

Example - Goal Programming

Find a solution for the following goal programming problem, by using a Lexicographic linear programming approach. Assume that Goal 1 is of more importance to the decision maker than Goal 2.

$$\begin{array}{ll} \text{Goal 1:} & x_1 + 2x_2 \geq 10 \\ \text{Goal 2:} & 3x_1 + x_2 \geq 8 \\ \text{s.t.} & 2x_1 - x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{array}$$

The Lexicographic linear programming problem

$$\begin{aligned} & L- \min\{s_1^-, s_2^-\} \\ x_1 + 2x_2 + s_1^- - s_1^+ &= 10 \\ 3x_1 + x_2 + s_2^- - s_2^+ &= 8 \\ 2x_1 - x_2 &\leq 40 \\ x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+ &\geq 0 \end{aligned}$$

The Lexicographic linear programming problem

$$\begin{array}{ll} \text{L- } \min\{s_1^-, s_2^-\} & \\ x_1 + 2x_2 + s_1^- - s_1^+ & = 10 \\ 3x_1 + x_2 + s_2^- - s_2^+ & = 8 \\ 2x_1 - x_2 & \leq 40 \\ x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+ & \geq 0 \end{array}$$

The Lexicographic linear programming problem

$$\begin{aligned} &L- \min\{s_1^-\} \\ &x_1 + 2x_2 + s_1^- - s_1^+ = 10 \\ &2x_1 - x_2 \leq 40 \\ &x_1, x_2, s_1^-, s_1^+ \geq 0 \end{aligned}$$

Solution: $x_2 = 5$ and $s_1^- = 0$. There are multiple solutions, and Goal 1 is met. Since there is “tie”, we proceed to solve the Lexicographic linear programming problem using Goal 2.

The Lexicographic linear programming problem

$$\begin{array}{rcl} \text{L- } \min\{s_2^-\} & & \\ x_1 + 2x_2 + s_1^+ & = & 10 \\ 3x_1 + x_2 + s_2^- - s_2^+ & = & 8 \\ 2x_1 - x_2 & \leq & 40 \\ x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+ & \geq & 0 \end{array}$$

Solution: $x_1 = 6/5$, $x_2 = 22/5$ and $s_2^- = 0$. Goal 2 is also met.

Example continued

Show, by drawing a graph, that the solution found lies in the interior of the feasible region given by the constraints.