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Deancorp produces sausage by blending beaf head, pork chuck, mutton, and water. The cost per pound, fat per pound, and protein per pound for these ingredients are given below:

	Head	Chuck	Mutton	Moisture
Fat (per lb)	0.05	0.24	0.11	0
Protein (per lb)	0.20	0.26	0.08	0
Cost (in cents)	12	9	8	0

Deancorp needs to produce 100 lb of sausage and has set the following goals, listed in order of priority.

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Goal 1: Sausage should consist of at least 15% protein.
Goal 2: Sausage should consist of at most 8% fat.
Goal 1: Cost per pound of sausage should not exceed 8 cents.

Formulate a Goal Programming model for the problem.

A Goal Programming Model

Let:

x_1 be amount of beef head (in pounds) used,

x_2 be the amount of pork chuck (in pounds) used,

x_3 be the amount of mutton (in pounds) used, and

x_4 be the amount of water (jn pounds) used.

$$
\begin{array}{rcll}
\text{Goal 1:} & 0.2x_1 + 0.26x_2 + 0.08x_3 & \geq & 15 \\
\text{Goal 2:} & 0.05x_1 + 0.24x_2 + 0.11x_3 & \leq & 8 \\
\text{Goal 3:} & 12x_1 + 9x_2 + 8x_3 & \leq & 80 \\
\text{s.t.} & x_1 + x_2 + x_3 + x_4 & = & 100 \\
 & x_1, x_2 & \geq & 0
\end{array}
$$

The Lexicographic linear programming problem

L- $\min\{s_1^-, s_2^-, s_3^-\}$

$$
\begin{array}{lcl}
0.2x_1 + 0.26x_2 + 0.08x_3 + s_1^- - s_1^+ & = & 15 \\
0.05x_1 + 0.24x_2 + 0.11x_3 \qquad + s_2^- & = & 8 \\
12x_1 + 9x_2 + 8x_3 \qquad + s_3^- & = & 80 \\
x_1 + x_2 + x_3 + x_4 & = & 100 \\
x_1, x_2, s_1^-, s_1^+, s_2^-, s_3^- & \geq & 0
\end{array}
$$

The Lexicographic linear programming problem

$$
\begin{array}{lcl}
\text{L- } \min\{s_1^-\} & & \\
0.2x_1 + 0.26x_2 + 0.08x_3 + s_1^- - s_1^+ & = & 15 \\
x_1 + x_2 + x_3 + x_4 & = & 100 \\
x_1, x_2, s_1^-, s_1^+ & \geq & 0
\end{array}
$$

Solution: $x_1 = 750/13$, $x_4 = 550/13$, and $s_1^- = 0$. There are multiple solutions, (because there are more zero reduced cost columns than number of rows), and Goal 1 is met. Since there is "tie", we proceed to solve the Lexicographic linear programming problem using Goal 2.

The Lexicographic linear programming problem

$$
\begin{array}{lll}
\text{L- } \min\{s_2^-\} & & \\
0.2x_1 + 0.26x_2 + 0.08x_3 - s_1^+ & & = 15 \\
0.05x_1 + 0.24x_2 + 0.11x_3 & +s_2^- & = 8 \\
x_1 + x_2 + x_3 + x_4 & & = 100 \\
x_1, x_2, s_1^+, s_2^- & & \geq 0
\end{array}
$$

Solution: $x_1 = 304/7$, $x_2 = 170/7$, $x_4 = 226/7$, and $s_2^- = 0$. Goal 2 is met, and there are multiple solutions, hence we move to optimise with respect to Goal 3.

Example continued

L- $\min\{s_3^-\}$

$$\begin{array}{llcl}
0.2x_1 + 0.26x_2 + 0.08x_3 - s_1^+ & & = & 15 \\
0.05x_1 + 0.24x_2 + 0.11x_3 & & = & 8 \\
12x_1 + 9x_2 + 8x_3 & +s_3^- & = & 80 \\
x_1 + x_2 + x_3 + x_4 & & = & 100 \\
x_1, x_2, s_1^+, s_3^- & & \geq & 0
\end{array}$$

Solution: Problem infeasible. Meaning, if we attemp to keep Goal 1 and Goal 2 satisfied at their optimal level, the problem becomes infeasible when Goal 3 is added in. Comment, if the constraint, $x_1 + x_2 + x_3 + x_4 = 100$, had been an inequality constraint, then feasible solution might be easier to obtain.