620-262 DECISION MAKING 2003

SOLUTIONS TO ASSIGNMENTS 1 → 5

ASSIGNMENT 1

Q1.
$$\begin{bmatrix} (16,64) & (32,48) & (39,41) \\ (41,39) & (47,33) & (56,24) \\ (29,51) & (50,30) & (26,54) \end{bmatrix}$$

Constant Sum Game : Total Payoff $= 80$

$80/2 = 40$ $\therefore$ Subtract 40 from each entry:

$$\begin{bmatrix} (-24,24) & (-8,8) & (-1,1) \\ (1,-1) & (7,-7) & (16,-16) \\ (-11,11) & (10,-10) & (-14,14) \end{bmatrix}$$

Simplify by taking Player I's point of view:

$$\begin{bmatrix} -24 & -8 & -1 \\ 1 & 7 & 16 \\ -11 & 10 & -14 \end{bmatrix}$$

Solve this zero-sum game:

- Determine the security levels for each strategy for each player
- Calculate L and U
- Since $L = U$, (a_2, A_1) is a saddle point

$\therefore$ Player I uses strategy 2 and Player II uses strategy 1.

The value of the game is $40 + 1 = 41$ (for Player I) or $(41, 39)$

Q2.

$$\begin{bmatrix} 3 & 4 & 5 \\ 9 & 7 & q \\ 4 & p & 6 \end{bmatrix}$$

1. For (a_2, A_2) to be a saddle point, 7 must be the smallest entry in row 2 and the largest entry in column 2.

$\therefore \min\{9, 7, q\} = 7$ AND $\max\{4, 7, p\} = 7$

$\Rightarrow \underline{q \geq 7}$ $\Rightarrow \underline{p \leq 7}$

2. (i) For (a_3, A_2) to be a saddle point, p must be the smallest entry in row 3 and the largest entry in column 2

$\therefore \min\{4, p, 6\} = p$ AND $\max\{4, 7, p\} = p$

$\Rightarrow \underline{p \leq 4}$ $\Rightarrow \underline{p \geq 7}$

$\therefore$ It is impossible for (a_3, A_2) to be a saddle point.

(ii) For (a_2, A_3) to be a saddle point, q must be the smallest entry in row 2 and the largest entry in column 3

$\therefore \min\{9, 7, q\} = q$ AND $\max\{5, q, 6\} = q$

$\Rightarrow \underline{q \leq 7}$ $\Rightarrow \underline{q \geq 6}$

$\therefore (a_2, A_3)$ is a saddle point for $\underline{6 \leq q \leq 7}$

From (i) we have (a_2, A_2) is a saddle point for $q \geq 7$.

Hence if $\underline{q = 7}$ both (a_2, A_2) and (a_2, A_3) are saddle points.