AssSol 10.pdf

ASSIGNMENT 10

We need to solve the problem

$$z^*(\beta) = \max \{3x_1 + 5x_2 + \beta(2x_1 - x_2)\}$$

ie

$$z^*(\beta) = \max \{(3+2\beta)x_1 + (5-\beta)x_2\}$$

s.t.

$$\begin{aligned} x_1 \quad\quad\quad &\leq 6 \\ x_1 + x_2 &\leq 12 \\ 3x_1 + 2x_2 &\leq 20 \\ x_1, x_2 &\geqslant 0 \\ \beta &> 0 \end{aligned}$$

We are given that the final tableau for $\beta = 0$ is

	x_1	x_2	x_3	x_4	x_5	RHS
x_3	1	0	1	0	0	6
x_4	$-1/2$	0	0	1	$-1/2$	2
x_2	$3/2$	1	0	0	$1/2$	10
z	$9/2$	0	0	0	$5/2$	50

From this we have $\quad x^*(0) = (0, 10)$

$\quad\quad\quad\quad\quad\quad\quad\quad z^*(0) = 50$

STEP 2: RANGE ANALYSIS

We determine the critical value of β by introducing it into the z-row. Since

$$z(\beta) = (3+2\beta)x_1 + (5-\beta)x_2$$

we have to add -2β to the reduced cost of x_1 and $+\beta$ to the reduced cost of x_2.

z	$9/2 - 2\beta$	β	0	0	$5/2$	50

We need to restore canonical form.

$$R_4' = R_4 - \beta R_3$$

③		x_1	x_2	x_3	x_4	x_5	RHS	② RATIO
$R_1' = R_1$	x_3	$\boxed{1}$	0	1	0	0	6	6 ← SMALLEST +ve
$R_2' = R_2 + \frac{1}{2}R_1'$	x_4	$-\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	2	–
$R_3' = R_3 - \frac{3}{2}R_1'$	x_2	$\frac{3}{2}$	1	0	0	$\frac{1}{2}$	10	$\frac{20}{3} = 6\frac{2}{3}$
$R_4'' = R_4' - \frac{1}{2}(9-7\beta)R_1'$	z	$\frac{1}{2}(9-7\beta)$	0	0	0	$\frac{1}{2}(5-\beta)$	$50-10\beta$	
		①↑						

This tableau is optimal if

$$\frac{1}{2}(9-7\beta) \geq 0 \qquad \left(\text{and } \frac{1}{2}(5-\beta) \geq 0 \right.$$

ie

$$0 < \beta \leq \frac{9}{7} \qquad \left. \text{and } 0 \leq \beta \leq 5 \right)$$

$$\therefore \; x^*(\beta) = (0, 10) \qquad z^*(\beta) = 50 - 10\beta, \qquad 0 < \beta \leq \frac{9}{7}$$

STEP 4: ITERATION

The critical value of β is generated by x_1, so we select x_1 as the new basic variable.

③		x_1	x_2	x_3	x_4	x_5	RHS	② RATIO
$R_3' = 2R_3$	x_1	1	0	1	0	0	6	–
$R_1' = R_1$	x_4	0	0	$\frac{1}{2}$	1	$-\frac{1}{2}$	5	–
$R_2' = R_2 + \frac{1}{2}R_3'$	x_2	0	1	$-\frac{3}{2}$	0	$\boxed{\frac{1}{2}}$	1	2 ← SMALLEST +ve
$R_4' = R_4 - \frac{1}{2}(5-\beta)R_3'$	z	0	0	$-\frac{1}{2}(9-7\beta)$	0	$\frac{1}{2}(5-\beta)$	$23 + 11\beta$	
						①↑		

This tableau is optimal if.

$$\frac{1}{2}(5-\beta) \geq 0 \quad \text{and} \quad -\frac{1}{2}(9-7\beta) \geq 0$$

ie

$$\frac{9}{7} \leq \beta \leq 5$$

$\therefore \; x^*(\beta) = (6,1) \quad z^*(\beta) = 23 + 11\beta, \quad 9/7 \leq \beta \leq 5$

STEP 4: ITERATION

The critical value of β is generated by x_5, so we select x_5 as the new basic variable.

	x_1	x_2	x_3	x_4	x_5	RHS
x_1	1	0	1	0	0	6
x_4	0	1	-1	1	0	6
x_5	0	2	-3	0	1	2
z	0	$\beta-5$	$3+2\beta$	0	0	$18+12\beta$

This tableau is optimal if $\beta \geq 5$

$\therefore \; x^*(\beta) = (6,0) \quad z^*(\beta) = 18 + 12\beta, \quad \beta \geq 5$

$\therefore$ The Pareto extreme points are

$\underline{x} = (0,10), \; (6,1), \; (6,0)$

and the corresponding objective values are

$\underline{z} = (50,-10), \; (23,11), \; (18,12)$