AssSol9.pdf

# ASSIGNMENT 9

The lexicographic linear programming problem is:

$$L\text{-min}\ (s_1^-, s_2^-)$$
s.t.
$$\begin{aligned}
2x_1 + x_2 + s_1^- - s_1^+ &= 14 \\
x_1 - x_2 + s_2^- - s_2^+ &= 6 \\
x_1 + x_2 &\leq 10 \\
x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+ &\geq 0
\end{aligned}$$

Since Goal 1 is the most important, we must first solve the problem:

$$L\text{-min}\ (s_1^-)$$
s.t.
$$\begin{aligned}
2x_1 + x_2 + s_1^- - s_1^+ &= 14 \\
x_1 + x_2 &\leq 10 \\
x_1, x_2, s_1^-, s_1^+ &\geq 0
\end{aligned}$$

Since the first constraint is an equality constraint, we need to add an artificial variable and use the two-phase method. Constraint 2 requires the addition of a slack variable.

| | $x_1$ | $x_2$ | $s_1^-$ | $s_1^+$ | $a_1$ | $x_3$ | RHS |
|---|---|---|---|---|---|---|---|
| $a_1$ | 2 | 1 | 1 | −1 | 1 | 0 | 14 |
| $x_3$ | 1 | 1 | 0 | 0 | 0 | 1 | 10 |
| w | 0 | 0 | 0 | 0 | −1 | 0 | 0 |

Restore canonical form (ie make the w-row of the basic variable $a_1$ equal to zero).

$$R_3' = R_3 + R_1$$

③

$R_1' = \frac{1}{2}R_1$

$R_2' = R_2 - R_1'$

$R_3' = R_3 - 2R_1'$

| | $x_1$ | $x_2$ | $s_1^-$ | $s_1^+$ | $\underline{a}_1$ | $x_3$ | RHS | ② RATIO |
|---|---|---|---|---|---|---|---|---|
| $\underline{a}_1$ | $\boxed{2}$ | 1 | 1 | −1 | 1 | 0 | 14 | 7 ← SMALLEST +ve |
| $x_3$ | 1 | 1 | 0 | 0 | 0 | 1 | 10 | 10 |
| $w$ | 2 | 1 | 1 | −1 | 0 | 0 | 14 | |

① MOST +ve

| | $x_1$ | $x_2$ | $s_1^-$ | $s_1^+$ | $\underline{a}_1$ | $x_3$ | RHS |
|---|---|---|---|---|---|---|---|
| $x_1$ | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 7 |
| $x_3$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1 | 3 |
| $w$ | 0 | 0 | 0 | 0 | −1 | 0 | 0 |

Since the artificial variable $\underline{a}_1$ is out of the basis (ie $\underline{a}_1 = 0$) we may now move to Phase 2.
Restore the original objective function.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $z_1$ | 0 | 0 | −1 | 0 | 0 | 0 | 0 |

This tableau is already in canonical form, and there are no non-basic variables with positive reduced costs.

∴ we STOP → this is the final tableau.

Since $s_1^-$ is non-basic, its value is 0, and we have the minimum value for $z_1$ (ie $z_1 = 0$ with $s_1 = 0$).

There are multiple optimal solutions since the non-basic variables $x_2$ and $s_1^+$ have reduced costs of zero.

Since there is a "tie", we proceed to solve the Lexicographic linear programming problem for Goal 2.

The problem for Goal 2 is:

$$L\text{-}\min\ (s_2^-)$$

s.t.

$$\begin{aligned}
2x_1 + x_2 - s_1^+ \qquad\qquad &= 14\\
x_1 - x_2 \qquad + s_2^- - s_2^+ &= 6\\
x_1 + x_2 \qquad\qquad &\le 10\\
x_1,\ x_2,\ s_1^+,\ s_2^-,\ s_2^+ &\ge 0\\
s_1^- &= 0
\end{aligned}$$

Since the first two constraints are equality constraints, we need to add an artificial variable for each and use the two-phase method. Constraint 3 requires the addition of a slack variable.

③

| | | $x_1$ | $x_2$ | $s_1^+$ | $s_2^-$ | $s_2^+$ | $a_1$ | $a_2$ | $x_3$ | RHS | ② RATIO |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_2' = R_2$ | $a_1$ | 2 | 1 | −1 | 0 | 0 | 1 | 0 | 0 | 14 | 7 |
| $R_1' = R_1 - 2R_2'$ | $a_2$ | $\boxed{1}$ | −1 | 0 | 1 | −1 | 0 | 1 | 0 | 6 | 6 ← SMALLEST +ve |
| $R_3' = R_3 - R_1'$ | $x_3$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 10 | 10 |
| | w | 0 | 0 | 0 | 0 | 0 | −1 | −1 | 0 | 0 | |

Restore canonical form: $R_4' = R_4 + R_1 + R_2$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_4'' = R_4' - 3R_2'$ | w | 3 | 0 | −1 | 1 | −1 | 0 | 0 | 0 | 20 |

↑ ① MOST +ve

③

| | | $x_1$ | $x_2$ | $s_1^+$ | $s_2^-$ | $s_2^+$ | $a_1$ | $a_2$ | $x_3$ | RHS | ② RATIO |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_1' = \frac{1}{3}R_1$ | $a_1$ | 0 | $\boxed{3}$ | −1 | −2 | 2 | 1 | −2 | 0 | 2 | 2/3 ← SMALLEST +ve |
| $R_2' = R_2 + R_1'$ | $x_1$ | 1 | −1 | 0 | 1 | −1 | 0 | 1 | 0 | 6 | − |
| $R_3' = R_3 - 2R_1'$ | $x_3$ | 0 | 2 | 0 | −1 | 1 | 0 | −1 | 1 | 4 | 2 |
| $R_4' = R_4 - 3R_1'$ | w | 0 | 3 | −1 | −2 | 2 | 0 | −3 | 0 | 2 | |

↑ ① MOST +ve

| | $x_1$ | $x_2$ | $s_1^+$ | $s_2^-$ | $s_2^+$ | $a_1$ | $a_2$ | $x_3$ | RHS |
|---|---|---|---|---|---|---|---|---|---|
| $x_2$ | 0 | 1 | $-1/3$ | $-2/3$ | $2/3$ | $1/3$ | $-2/3$ | 0 | $2/3$ |
| $x_1$ | 1 | 0 | $-1/3$ | $1/3$ | $-1/3$ | $1/3$ | $1/3$ | 0 | $20/3$ |
| $x_3$ | 0 | 0 | $2/3$ | $1/3$ | $-1/3$ | $-2/3$ | $1/3$ | 1 | $8/3$ |
| $w$ | 0 | 0 | 0 | 0 | 0 | $-1$ | $-1$ | 0 | 0 |

Since the artificial variables $a_1$ and $a_2$ are out of the basis (ie $a_1 = a_2 = 0$) we may now move to Phase 2.

Restore the original objective function.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $z_2$ | 0 | 0 | 0 | $-1$ | 0 | 0 | 0 | 0 | 0 |

This tableau is already in canonical form, and there are no non-basic variables with positive reduced costs.

$\therefore$ we STOP $\rightarrow$ this is the final tableau.

Since $s_2^-$ is non-basic, its value is 0, and we have the minimum value for $z_2$ (ie $z_2 = 0$ with $s_2^- = 0$).

There are multiple optimal solutions since the non-basic variables $s_1^+$ and $s_2^+$ have reduced costs of zero.

However, since there are no more goals to consider, we have our final solution.

Our solution is: $x_1 = 20/3 = 6\,2/3$

$x_2 = 2/3$

Goal 1 is satisfied.

Goal 2 is satisfied.