Example - Advertising Campaign

Liam Merlot, the owner of a winery can use advertisement through one of three media: radio, TV, or newspaper. The weekly costs of advertisement in the three media are estimated at $200, $900, and $300 respectively. The company can classify its sales volume during each week as (1) fair, (2), good, or (3) excellent. The transition probabilities associated with each advertisement medium are given in the next few slides.

A return value has been determined for each pair of states, and each choice of media.

Problem: Help Liam plan an advertising policy such that the return value is maximised. What medium should he use when the sales is fair, what medium to use when the sales is good, and what to use when the sales is excellent?

Example - Advertising Campaign

Transition matrix for Radio:

	$fair$	$good$	$excellent$
$fair$	0.4	0.5	0.1
$good$	0.1	0.7	0.2
$excellent$	0.1	0.2	0.7

For example, if sales volumes are good, and the company chooses to advertise in radio, there is a 20% chance that sales will be excellent in the next period.

Example - Advertising Campaign

Transition matrix for TV:

	$fair$	$good$	$excellent$
$fair$	0.7	0.2	0.1
$good$	0.3	0.6	0.1
$excellent$	0.1	0.7	0.2

Transition matrix for Newspaper:

	$fair$	$good$	$excellent$
$fair$	0.2	0.5	0.3
$good$	0	0.7	0.3
$excellent$	0	0.2	0.8

Example - Advertising Campaign

The weekly returns (in thousands of dollars) for Radio is:

	$fair$	$good$	$excellent$
$fair$	400	520	600
$good$	300	400	700
$excellent$	200	250	500

For example, the entry in the 1st row and 2nd column of the return matrix is 520, so if in a fair state the company chooses to advertise in radio, and the subsequent state is good, return to the company will be $520 thousand.

Example - Advertising Campaign

The weekly returns (in thousands of dollars) for TV is:

	$fair$	$good$	$excellent$
$fair$	1000	1300	1600
$good$	800	1000	1700
$excellent$	600	700	1100

The weekly returns (in thousands of dollars) for Newspaper is:

	$fair$	$good$	$excellent$
$fair$	400	530	710
$good$	350	450	800
$excellent$	250	400	650

Example - Advertising Campaign

State the generic form of the value determination equations.

Let:
-$S = \{F, G, E\}$ be the set of states of the MD
- $D(i) = \{R, T, N\}$ be the set of decisions with R representing radion, T representing TV, and N representing newspaper, for all $i \in S$.
- r_{id} for $i \in \{F, G, E\}$ be the return value for state i using decision d,
- c_d for $d \in \{R, T, N\}$ be the costs of advertising using medium d,
- $p(j|i, d)$ be the probability that next period's state is j, given that the cu rrent period begins in state i and decision j is chosen
- V_i for $i \in \{F, G, E\}$ be the expected discounted rate given that at the be giinning of period 1, the state is i, and
- β the discount factor.

Example - Advertising Campaign

The generic form of the value determination equations is given by:

$$V_i = \max_{d \in D(i)} \{r_{id} + \beta \sum_{j \in S} p(j|i,d) V_j\},$$

$$\text{where } r_{id} = \sum_{j \in S} p(j|i,d) r_{ij}^d - c_d,$$

for r_{ij}^d the return value if current state is i, next state is state j, and decision is d.

For example,

$$\begin{aligned} r_{FR} &= (0.4, 0.5, 0.1) \begin{bmatrix} 400 \\ 520 \\ 600 \end{bmatrix} - 200 \\ &= 160 + 260 + 60 - 200 \\ &= 280. \end{aligned}$$

and

$$\begin{aligned} r_{GR} &= (0.1, 0.7, 0.2) \begin{bmatrix} 300 \\ 400 \\ 700 \end{bmatrix} - 200 \\ &= 30 + 280 + 140 - 200 \\ &= 250. \end{aligned}$$

Example - Advertising Campaign

State the linear programming model for the optimisation of advertisement policy for an infinite planning horizon assuming a discount factor $\beta = 0.9$.

$$
\begin{array}{rcll}
\min z & = & V_F + V_G + V_E & \\
V_F & \geq & 280 + 0.36V_F + 0.45V_G + 0.09V_E & (F,R) \\
V_F & \geq & 220 + 0.63V_F + 0.18V_G + 0.09V_E & (F,T) \\
V_F & \geq & 258 + 0.18V_F + 0.45V_G + 0.27V_E & (F,N) \\
V_G & \geq & 250 + 0.09V_F + 0.63V_G + 0.18V_E & (G,R) \\
V_G & \geq & 110 + 0.27V_F + 0.54V_G + 0.09V_E & (G,T) \\
V_G & \geq & 255 + 0.63V_G + 0.27V_E & (G,N) \\
V_E & \geq & 220 + 0.09V_F + 0.18V_G + 0.63V_E & (E,R) \\
V_E & \geq & -130 + 0.09V_F + 0.63V_G + 0.18V_E & (E,T) \\
V_E & \geq & 300 + 0.18V_G + 0.72V_E & (E,N) \\
 & & V_F, V_G, V_E \geq 0 &
\end{array}
$$

Example - Advertising Campaign

Results: By solving the LP model, we observed that the constraints that represent (F, R), (G, N), and (E, N) are satisfied at equality.

Conclusions: This indicates that the optimal decision for the problem is that when the company's sales volume is fair, we should advertise on radio, and when the company's sales volume is either good or excellent, we should advertise on newspaper.