Solutions to HW-1 problems.

1 (a) For binomial markets, NA $\Leftrightarrow$ $d < 1+r < u$ (see slide 30). We have $d = S_1(\omega_2)/S_0 = 8/9 < 1+r = 10/9 < u = S_1(\omega_1)/S_0 = 4/3 = 12/9$.

(b) Since $p^* = \frac{1+r-d}{u-d} = \frac{10/9 - 8/9}{12/9 - 8/9} = \frac{1}{2}$, $1-p^* = \frac{1}{2}$,

the claim value is

$$X^* = \frac{1}{1+r} E^* X = \frac{9}{10} \times \left[\frac{1}{2}\times 7 + \frac{1}{2}\times 2\right] = \frac{81}{20} \quad (= 4.05).$$

(c)
$$\Delta = \frac{X_u - X_d}{uS_0 - dS_0} = \frac{7-2}{\frac{4}{9}\times 5} = \frac{9}{4} \quad (= 2.25),$$

$$b = \frac{uX_d - dX_u}{(1+r)(u-d)} = \frac{\frac{12}{9}\times 2 - \frac{8}{9}\times 7}{\frac{10}{9}\times\frac{4}{9}} = -\frac{36}{5} \quad (= -7.2),$$

so the replicating portfolio $(\Delta, b) = \left(\frac{9}{4}, -\frac{36}{5}\right)$.

Its time $t=0$ value:

$$V_0 = \Delta S_0 + b = \frac{9}{4}\times 5 + \left(-\frac{36}{5}\right) = \frac{81}{20}, \text{ ok,}$$

time $t=1$ value:

$$V_1 = \Delta S_1 + (1+r)b = \begin{cases} \frac{9}{4}\times\frac{20}{3} - \frac{36}{5}\times\frac{10}{9} = 7 & \text{if } \omega = \omega_1 \\ \frac{9}{4}\times\frac{40}{9} - \frac{36}{5}\times\frac{10}{9} = 2 & \text{if } \omega = \omega_2 \end{cases}, \text{ ok.}$$

2 (a) From slide 23, 15 (or from 27 & PCP 34),

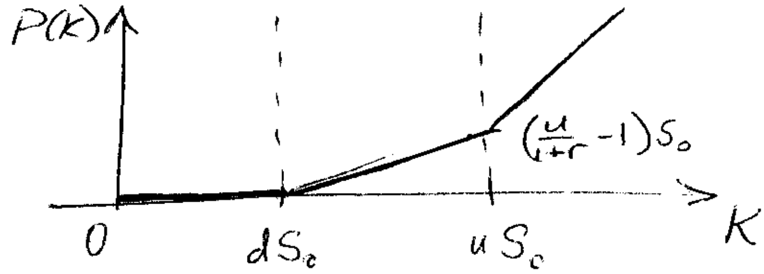
$$P(K) = \frac{1}{1+r}\left[p^*(S_0 u - K)^- + (1-p^*)(dS_0 - K)^-\right].$$

So if $K \le dS_0$, both $(\cdot)^- = 0$ so that $P(K) = 0$; if $dS_0 \le K \le uS_0$, then

$P(K) = \frac{1-p^*}{1+r}(K - dS_0)$; if $K \ge uS_0$, then

$$P(K) = \frac{1}{1+r}\left[p^*(K-uS_0) + (1-p^*)(K-dS_0)\right] = \frac{K}{1+r} - S_0$$

(you could also use the result of tutorial problem 2 & the PCP).



(b) $u = \frac{4.6}{4} = 1.15$, $d = \frac{3.6}{4} = 0.9$, so $d < 1+r = 1.05 < u$, Ok.

$p^* = \frac{1+r-d}{u-d} = \frac{0.15}{0.25} = \frac{3}{5}$, $1 - p^* = \frac{2}{5}$, so

$$P = \frac{1}{1.05}\left[0.6 \times \underbrace{(4.6-3.8)^-}_{=0} + 0.4 \times \underbrace{(3.6-3.8)^-}_{=0.2}\right] = \frac{8}{105} \; (\approx 0.076)$$

(c) From the PCP (slide 34),

$$C = S_0 + P - \frac{K}{1+r} = 4 + \frac{8}{105} - \frac{3.8}{1.05} = \frac{48}{105} = \frac{16}{35} \; (\approx 0.457)$$