Please find attached the *Plagiarism Declaration Sheet* form for Semester 2, 2009. The sheet **must** be completed, signed, dated and attached at the front of your homework. Otherwise your work will not be marked.

Solutions to the *homework problems* are to be left in the 620-302 assignment box (#**181**) on the ground floor in the Richard Berry Building (north entrance). **Don't forget** to print your name, student ID, the subject name and code and your lecturer's name (K. Borovkov) on the first page of your solutions! All homework problems should be attempted. Only one (randomly chosen) of them will be marked. All material handed in must be on A4 size paper. Material on different sized paper will not be marked. The form and neatness of work can be considered in marking. Working and/or reasoning **must** be given to obtain full credit. The submission deadline is **5pm on Monday, 10 August 2009**.

Tutorial Problems

In what follows, "call" ("put") means a European call (put) option.

1. Let X and Y be two contingent claims (possibly on different underlying stocks and with different expiry dates) in an arbitrage-free market. Denote by X^* and Y^* the arbitrage-free ("fair") prices of the respective claims at time $t = 0$. Using an arbitrage-based argument, show that the arbitrage-free price of the claim $X + Y$ at time $t = 0$ is equal to $X^* + Y^*$.

2. For a single-period binomial market, sketch the graph of the call price $C = C(K)$ as a function of strike K. Which features of this function you think will be common to the call prices in more general markets as well?

3. A *straddle* is a portfolio consisting of a call and a put on the same underlying stock, with the same strike and same expiry. Plot the graph of the payoff function (of the terminal stock price) of the straddle (on one share of a stock).

4. A *bull spread* is a portfolio formed by buying a call with strike price K_1 and selling a call with strike price $K_2 > K_1$. Both calls are on the same underlying stock and have the same expiry date T. A bull spread is obviously a contingent claim.

   (a) Plot the payoff function of a bull spread.

   (b) What is cheaper: a bull spread or a call with the strike K_1 (on the same underlying stock, with the same expiry date)? Explain.

   (c) Consider a single-period binomial market. Assume that the current stock price is $S_0 = 5$, the possible values of S_1 are 4 and 6, $r = 0.1$, $K_1 = 3$ and $K_2 = 5$. Price the bull spread using the risk-neutral valuation ("fair price") formula on lecture slide 23.

(d) Under the assumptions from part (c), construct a replicating portfolio for the bull spread and verify that the portfolio does replicate the claim.

5. Consider a single period "trinomial" market: $\Omega = \{d, m, u\}$ (i.e. at time $t = 1$ there are three possible states of the world now), with $S_1(d) = dS_0$, $S_1(m) = mS_0$ and $S_1(u) = uS_0$, where $0 < d < m < u$. The time $t = 0, 1$ bond prices are $B_0 = 1$ and $B_1 = 1 + r$, respectively, where $d < 1 + r < u$.

   (a) Use the No-arbitrage Theorem (slide 43) to verify (or disprove) that the market is arbitrage free.

   (b) Let X be a contingent claim, with some values $X_d = X(d)$, $X_m = X(m)$ and $X_u = X(u)$ for the possible states of the world in our model ($X_d < X_m < X_u$). Draw a diagram showing the set of all hedges (Δ, b) for the claim (similar to the one on slide 21 for the binomial market). Does there exist a perfect hedge (i.e. a replicating portfolio) for X?

Homework Problems

1. Consider a single-period binomial financial market with $\Omega = \{\omega_1, \omega_2\}$, with the current (at time $t = 0$) asset prices $S_0 = 5$ and $B_0 = 1$, and the terminal (at time $t = 1$) stock price $S_1(\omega_1) = 20/3$ and $S_1(\omega_2) = 40/9$. Assume the interest rate $r = 1/9$.

   (a) Show that the market is arbitrage-free.

   (b) Consider a contingent claim X with $X(\omega_1) = 7$ and $X(\omega_2) = 2$. Find the value of this claim at time $t = 0$.

   (c) Construct a replicating portfolio for the claim X and verify that its values at times $t = 0$ and $t = 1$ coincide with the claim price you found in part (b) and the claim value X, respectively.

2. For a single-period binomial market:

   (a) sketch the graph of the put price $P = P(K)$ as a function of strike K;

   (b) price a put with strike $K = 3.8$ on one share of the stock with current price $S_0 = 4$, if $r = 5\%$ and possible values of the time $t = 1$ stock price are $S_1 = 3.6$ and $S_1 = 4.6$;

   (c) find the time $t = 0$ value of the call with the same strike.