

Solutions to the *homework problems* are to be left in the 620-302 assignment box (#181) on the ground floor in the Richard Berry Building (north entrance). **Don't forget** to print your name, student ID, the subject name and code and your lecturer's name (K. Borovkov) on the first page of your solutions! All homework problems should be attempted. Only one (randomly chosen) of them will be marked. All material handed in must be on A4 size paper. Material on different sized paper will not be marked. The form and neatness of work can be considered in marking. Working and/or reasoning **must** be given to obtain full credit. The submission deadline is **5pm on Monday, 17 August 2009**.

### Tutorial Problems

1. Consider a single period trinomial market with two stocks (stock 1 and stock 2) and one bond with  $r = 0$ . Assume that  $\Omega = \{\omega_1, \omega_2, \omega_3\}$  (i.e. at time  $t = 1$  there are three possible states of the world now), with the initial (time  $t = 0$ ) stock prices  $S_0^1 = 4$  and  $S_0^2 = 5$  for stock 1 and stock 2, resp., and the time  $t = 1$  stock prices

$$\begin{aligned} S_1^1(\omega_1) &= 6, & S_1^2(\omega_1) &= 6; \\ S_1^1(\omega_2) &= 4, & S_1^2(\omega_2) &= 4; \\ S_1^1(\omega_3) &= 2, & S_1^2(\omega_3) &= 7. \end{aligned}$$

- (a) Depict the stock prices at the times  $t = 0$  and  $t = 1$  by points on the (stock 1 price, stock 2 price)-plane.
  - (b) Use the No-arbitrage Theorem (slide 43) to prove or disprove that the market is arbitrage free.
  - (c) Find the EMM  $\mathbf{P}^*$  and use the Completeness Theorem (slide 52) to prove or disprove that the market is complete.
  - (d) Price the call on (one share of) stock 1 with expiry  $T = 1$  and strike  $K = 5$ .
  - (e) Find the strategy replicating the call from part (d). Do we really need stock 2 to replicate the call on stock 1?
2. Consider a  $T = 2$  period binomial market with  $u = 1.75$ ,  $d = 0.5$  and  $r = 0.25$ . Assuming that the time  $t = 0$  stock price  $S_0 = 400$ ,
    - (a) use the diagram method (cf. slides 29 and 65–66) to price a put option with maturity  $T = 2$  and strike  $K = 450$ ;
    - (b) construct a replicating portfolio for this option;
    - (c) directly verify that the replicating strategy you found in part (b) is self-financing;
    - (d) price a call with the same maturity and strike price.
  3. It is important to know the impact of the underlying parameters (the current stock price  $S_0$ , volatility  $\sigma$ , interest rate  $r$ , time to maturity  $T$  and the strike  $K$ ) in the Black-Scholes formula on the prices of call and put options. The *sensitivites* of the option price with respect to the first four parameters are

called the *Greeks* (due to the use of Greek letters to denote them) and are used for hedging purposes. They are computed as partial derivatives of the option value with respect to the parameters (and can be calculated for more general claims/portfolios as well).

For the (European) call (see slide 79 for the Black-Scholes formula for the call value), compute:

- (a) delta  $\Delta = \frac{\partial C}{\partial S_0}$  (measures the change in the option value when the stock price is changing; also gives the number of shares in the replicating portfolio for the call option);
- (b) gamma  $\Gamma = \frac{\partial^2 C}{\partial S_0^2}$  (measures the sensitivity of the *hedging portfolio* for the call to the change of the stock price);
- (c) vega  $\mathcal{V} = \frac{\partial C}{\partial \sigma}$ .<sup>1</sup>
- (d) Can you find delta for the European put in the Black-Scholes framework without doing any computations? What can you say about the signs of the deltas for call/put options?

## Homework Problems

You may wish to use *MS Excel* or some other software package when solving the homework problems. But don't forget to show your working/reasoning!

1. Consider a four-period binomial financial market with  $u = 4/3$ ,  $d = 2/3$  and  $r = 0$  (i.e. the prices have already been discounted). Let the current (time  $t = 0$ ) stock price be  $S_0 = 81$ . Use the diagram method (cf. slides 29 and 65–66) to price a call with maturity  $T = 4$  and strike  $K = 120$ .
2. (a) On 28 July 2000 ( $t = 0$ ), the BHP stock price was  $S_0 = 18.50$ . Supposing the (continuously compounded) interest rate  $r = 0.062$  (can be worked out), use the Black-Scholes formula to find the price of the call option with maturity one month later ( $T = 1/12$  year) and strike  $K = 18.50$  (assume the volatility value  $\sigma = 0.2$ ).
- (b) The real-life buy/sell prices of the option were 0.57/0.63, resp. Taking the mean of these two values as the market price of the option, find the volatility value such that the Black-Scholes formula gives the closest value to the market price (you may wish to try  $\sigma = 0.23, 0.24, 0.25$  etc). This value is called the *implied volatility*.

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<sup>1</sup>This is not a Greek letter! The vega is an exception: for some unclear reasons, it was named after the brightest star in the constellation Lyra (note that this sensitivity is also denoted occasionally by the Greek letters epsilon, kappa, tau, lambda and zeta...). The use of *vega* has probably persisted due to its starting with a *v* (like the *volatility* whose influence on the prices it refers to). The two remaining Greeks are: theta  $\Theta = \partial C / \partial T$  and rho  $\rho = \partial C / \partial r$ .