
620-321 Algebra, Semester 1, 2009
Answers to Problem Sheet 7

1. There are 3 abelian groups of order 136.

Noting that $136 = 2^3 \times 17$ and considering the possible primary decompositions we obtain the following:

$$G_1 = \mathbb{Z}_8 \oplus \mathbb{Z}_{17} \quad G_2 = \mathbb{Z}_2 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_{17} \quad G_3 = \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_{17}$$

The corresponding invariant factor decompositions are:

$$G_1 \cong \mathbb{Z}_{136} \quad G_2 \cong \mathbb{Z}_2 \oplus \mathbb{Z}_{68} \quad G_3 \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_{34}$$

2. The invariant factors are 2, 300, 900.

3. $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$

4. The invariant factor matrices are as follows:

$$\begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & x+1 & 0 \\ 0 & 0 & x^2-1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-x & 0 \\ 0 & 0 & x(1-x^2) \end{bmatrix}$$

5. $\mathbb{Z}[i]/(11-8i)$

6. (a) $C_4 \oplus \mathbb{Z}$

(b) $C_2 \oplus C_2$

(c) $C_2 \oplus \mathbb{Z}$

7. The invariant factors are 1, 3, 6 and so $M \cong C_3 \oplus C_6$.

8.

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -5 \end{bmatrix}$$

The eigenvalues of T are 3 (with algebraic multiplicity 3, and geometric multiplicity 1) and -5 (with algebraic multiplicity 5, and geometric multiplicity 2). T has 3 (linearly independent) eigenvectors. The characteristic polynomial is $\chi_T(x) = (x-3)^3(x+5)^5$. The minimal polynomial is $\mu_T(x) = (x-3)^3(x+5)^3$.