
620-321 Algebra, Semester 1, 2009
Answers to Problem Sheet 8

1. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.

$$\begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \quad \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \quad \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \quad \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \quad \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

3. Primary decomposition is:

$$M \cong \frac{\mathbb{Q}[x]}{(x+2)} \oplus \frac{\mathbb{Q}[x]}{(x-1)} \oplus \frac{\mathbb{Q}[x]}{(x-1)^3} \oplus \frac{\mathbb{Q}[x]}{(x^2+1)^2} \oplus \frac{\mathbb{Q}[x]}{(x^2+1)^2} \oplus \frac{\mathbb{Q}[x]}{(x^2+1)^4}$$

Invariant factor decomposition is:

$$M \cong \frac{\mathbb{Q}[x]}{(x^2+1)^2} \oplus \frac{\mathbb{Q}[x]}{(x^2+1)^2(x-1)} \oplus \frac{\mathbb{Q}[x]}{(x^2+1)^4(x-1)^3(x+2)}$$

M has dimension 8, and the minimal and characteristic polynomials are:

$$\mu_A(t) = (x^2+1)^4(x-1)^3(x+2), \quad \text{and} \quad \chi_A(t) = (x^2+1)^8(x-1)^4(x+2).$$

4. (a) $\text{irr}(a, \mathbb{Q}) = x^4 - 6x^2 + 3$ (b) $\text{irr}(a, \mathbb{Q}) = x^4 - \frac{2}{3}x^2 - \frac{62}{9}$ (c) $\text{irr}(a, \mathbb{Q}) = x^4 - 2x^2 + 9$

5. (a) The dimension is 2 and $\{1, i\}$ is a basis over \mathbb{R} .
 (b) The dimension over \mathbb{Q} is 4 and a basis is $\{1, \sqrt{2}, i, i\sqrt{2}\}$.
 (c) The dimension is 4 with basis $\{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\}$.
 (d) Dimension 4, basis $\{1, \sqrt{3}, i, i\sqrt{3}\}$.

6. The minimal polynomials are:

- (a) $x^3 - 2$
 (b) $x^4 - 10x^2 + 1$
 (c) $x^2 - x - 1$
 (d) $x^2 + x + 1$