

Problem Sheet 11

For each of the following polynomials $f \in \mathbb{Q}[x]$ calculate:

- (a) The size of the Galois group $G = G(K/\mathbb{Q})$, where K is the splitting field of f (over \mathbb{Q});
- (b) Identify the group G ;
- (c) List the correspondence between subgroups of G and intermediate fields L ,
 $\mathbb{Q} \subseteq L \subseteq K$.

1. $x^2 - 5x + 6$

2. $x^2 - 2$

3. $x^4 - x^2 - 2$

4. $x^3 - 7$ (Which subfields of K are Galois extensions of \mathbb{Q} ?)

5. $x^3 - 1$

6. $x^5 - 1$

For the following polynomials in $\mathbb{Q}[x]$, calculate the size of the Galois group $G = G(K/\mathbb{Q})$, where K is the splitting field of the polynomial. Also give an intermediate field L such that $\mathbb{Q} \subseteq L \subseteq K$ and L is *not* a Galois extension of \mathbb{Q} (if such exists).

7. $x^4 + 1$

8. $x^4 - 2$

Can you identify the Galois group in each case?