Departmen t of Mathematics and Statistics
620–342
Industrial and Applied Mathematics

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Problem Sheet 1.
Streamlines, particle paths and streamfunctions

Question 1

At time t, the velocity field $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$ is given by

$$u(x, y, t) = \frac{\alpha x}{1 + \alpha t}; v(x, y, t) = \beta$$

where α and β are constants.

i. Find the streamlines

ii. Find the path of the particle that was located at (X,Y) at t=0

iii. Verify from the particle path that

$$\frac{\partial \mathbf{u}}{\partial t} |_{\mathbf{R}} = \frac{D\mathbf{u}}{Dt}$$

Question 2

For the steady 2-dimensional flows

- $$u(x, y) = y; \quad v(x, y) = x$$

- $$u(x, y) = y; v(x, y) = -x$$

i. Find the curves describing the shape of the streamlines

ii. Find the particle paths in terms of (X,Y)

iii. Verify that

$$\frac{\partial \mathbf{u}}{\partial t} |_{\mathbf{R}} = \frac{D\mathbf{u}}{Dt}$$

For the second flow, find $\nabla \times \mathbf{u}$ and describe the flow.

Question 3

For the steady 3-dimensional flow

$$\mathbf{u}(x, y, z) = (x, y, -2z)$$

i. Find the particle paths in terms of (X,Y,Z)

ii. Verify that

$$\frac{\partial \mathbf{u}}{\partial t}|_{\mathbf{R}} = \frac{D\mathbf{u}}{Dt}$$

iii. Describe the flow

Question 4

Which of the flows above are incompressible?

For those that are plane flows, construct the streamfunction and hence sketch the streamlines.

Question 5

Show that any 2D flow of the form

$$u_\sigma = 0, u_\phi = f(\sigma)$$

is a possible incompressible flow and find the streamlines in plane polar coordinates for the special cases

i. $f(\sigma) = C\sigma$

ii. $f(\sigma) = \frac{D}{\sigma}$

In each case, calculate $\nabla \times \mathbf{u}$. Describe the flows.

Question 6

For the uniform stream

$$\mathbf{u} = U\mathbf{i}$$

find the streamfunction in both Cartesian coordinates and plane polar coordinates(viz.(σ, ϕ)).

Question 7

For the 2D flow represented by the streamfunction in plane polar coordinates

$$\psi = U\sigma \sin\phi - Ua^2 \sin\phi/\sigma$$

sketch the streamlines. In particular, describe the flow at large σ, find any stagnation points in the flow (points where $\mathbf{u} = \mathbf{0}$), describe the streamline $\psi = 0$, calculate $\nabla \times \mathbf{u}$ and describe what sort of flow it could possibly represent.

There is no need to consider the case $\sigma < a$.