Department of Mathematics and Statistics
620–342
Industrial and Applied Mathematics

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Problem Sheet 1. Some answers

Question 1

The particle paths are lines but the streamlines are not.

Question 2

For the first flow

- streamlines are hyperbolae
- $\mathbf{r}(t;\mathbf{X}) = (X\cosh t + Y\sinh t, X\sinh t + Y\cosh t)$

For the second

- streamlines are circles
- $\mathbf{r}(t;\mathbf{X}) = (X\cos t + Y\sin t, -X\sin t + Y\cos t)$
- $\nabla\times\mathbf{u} = -2\mathbf{k}$
- rigid body rotation about the z-axis

Question 3

$\mathbf{r}(t;\mathbf{X}) = (Xe^t, Ye^t, Ze^{-2t})$

Flow is axisymmetric about the z-axis but leaves the z-axis and tends to radial flow along the xy-plane — called *biaxial extensional flow*. Flow pushes material towards the xy plane but away from the z-axis.

Question 4

The second flow in Question 2 and the flow in Question 3 are incompressible.

In the first case, the streamfunction ψ is $\psi = (x^2+y^2)/2$

Question 5

i. $\psi = -\frac{C}{2}\sigma^2 + c$; $\nabla\times\mathbf{u} = 2C$; rigid body rotation

ii. $\psi = -D\log_e\sigma + c$;$\nabla\times\mathbf{u} = 0$; irrotational flow (a 'line vortex')

Question 6

$\psi = Uy = U\sigma\sin\varphi$

Question 7

- Flow tends to uniform flow at large σ
- streamline $\psi = 0$ is the circle $\sigma = a$
- stagnation points at $\sigma = a, \varphi = 0, \pi$
- $\nabla \times \mathbf{u} = 0$
- potential flow past an infinitely long cylinder (axis perpendicular to $\mathbf{U}$)