

Department of Mathematics and Statistics
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Industrial and Applied Mathematics

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Problem Sheet 3. Some answers

Question 1

The body floats if its mass equals the sum of the masses of the displaced fluids.

Question 2

- 10 metres
- 1.2 atmospheres

Question 3

The scale height H_s is about 8 km.

Question 4

The flow is not irrotational so we are not entitled to equate the Bernoulli head across different streamlines. So the assertion 'by Bernoulli' is wrong.

After integrating the Euler equations for the pressure (easy since we're given the velocity field) we get surfaces of constant pressure obeying

$$z = C + \frac{\Omega^2}{2g}(x^2 + y^2)$$

as we expect.

Question 5

Use the fact that $\nabla \times \mathbf{u} = 0$ and so $\mathbf{u} = \nabla\phi$ where ϕ is the velocity potential (*not* the azimuthal angle!!)

The choice of $F(t)$ can't matter since we could absorb it into ϕ but $\mathbf{u} = \nabla\phi$ wouldn't be affected.

Question 6

The term $\frac{1}{\rho}\nabla p$ in the usual derivation can be written as $\nabla F(\rho)$ where

$$\frac{dF}{d\rho} = \frac{1}{\rho} \frac{df}{d\rho}$$

Hence the head is modified to include $F(\rho)$ instead of $\frac{p}{\rho}$

For the particular case, $f(\rho) = k\rho^\gamma$,

$$F = \frac{\gamma}{\gamma - 1} \frac{p}{\rho} \approx 3.5 \frac{p}{\rho}$$

Question 7

Just express the velocity components in terms of Ψ and impose the irrotationality condition.

Question 8

This represents potential flow about an axisymmetric body whose surface is given by the equation $r = a \csc \frac{\theta}{2}$.
The pressure is given by

$$p = p_{\infty} - \frac{1}{2} \rho U^2 \left(2 \frac{a^2}{r^2} \cos \theta + \frac{a^4}{r^4} \right)$$