Departmen t of Mathematics and Statistics
620–342
Industrial and Applied Mathematics

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Problem Sheet 5. Some answers

Question 1

Although there are no viscous *forces* acting on the fluid there are still viscous *stresses* that act on the cylinder to produce the drag.

Question 2

The viscous normal stress is $\mu\frac{\partial w}{\partial z}$ if the normal is **k**. By continuity, this is -$\mu(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})$.
Then use the no-slip boundary condition.

Question 3

roughly: $4 \times 10^7; 10^{-3}; 10^{-3}; 20,000; 0.7$

Question 4

It helps to write the flow as the steady Couette flow plus a transient part. Then the transient part has homogeneous boundary conditions.

Separation of variables is immediate. Just choose the coefficients (using Fourier series) to make the initial value of the transient cancel the steady flow so that the total flow starts from rest.

$$u = U(1 - \frac{y}{d}) - \frac{2U}{\pi} \sum \frac{1}{n} \exp(-n^2\pi^2\nu t/d^2) \sin n\pi y/d$$

stress is $\mu\frac{\partial u}{\partial y}\mid_{y=0} = -\frac{\mu U}{d} - \frac{2\mu U}{d} \sum \exp(-n^2\pi^2\nu t/d^2)$
Notice that the transient stress is a *completely monotone* function of time
(a function f is completely monotone if $(-1)^n f^{(n)} \geq 0 \quad n = 1, 2, \cdots$)

Question 5

This is a straightforward generalization of steady Poiseuille flow through a circular pipe.

$$w = \frac{G}{4\mu}[(a^2 - \sigma^2) + 3a^2 \log(\sigma/a)/\log 2]$$

Volume flux is just $\int_a^{2a} 2\pi\sigma w \; d\sigma$
The shear stresses on the boundaries are just $\pm\mu\frac{dw}{d\sigma}$

Question 6

Start from Eq. 7.22 of the Notes (note the typo: $u_\phi = \Omega\sigma$ initially)

Separating variables produces exponentially decaying functions of time and Bessel functions of order 1 for the radial dependence.

Finding the coefficients to satisfy the initial condition as in 331 gives a ratio of integrals of Bessel functions. To get the final results, use various orthogonality conditions and recurrence relations for Bessel functions. See Eqs. 11.4.5, 11.3.20 and 9.1.27 in Abramowitz and Stegun, *Handbook of Mathematical Functions*