Departmen t of Mathematics and Statistics
620–342
Industrial and Applied Mathematics

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Problem Sheet 7. Some answers

Question 1

The angular eigenfunctions for the Laplacian in spherical coordinates satisifies the equation (see 331 Notes, p.100)

$$[(1-x^2)Y']' + n(n+1)Y = 0$$

which is Legendre's equation, having solutions the Legendre polynomials $P_n(x)$.

Take the derivative of the corresponding equation for E^2 (given in question), we get

$$[(1-x^2)\mathcal{Q}'']' + n(n+1)\mathcal{Q}' = 0$$

showing that

$$\frac{d\mathcal{Q}}{dx} = Y = P_n(x)$$

Hence

$$\mathcal{Q}_n = \int_c^x P_n(x')\ dx'$$

$c = -1$ because the requirement that $u_\theta = 0$ for $\theta = 0, \pi$ by axisymmetry means that $\mathcal{Q}(\cos\theta) = 0$ for $\theta = 0, \pi$.

Question 2

i. Just use $\mathbf{U} = \Omega\hat{\mathbf{k}} \times a\mathbf{e}_r$

ii.

iii.

Question 3

Question 4

Question 5

Question 6

Question 7