**Problem 1.** Consider a unimodal function \( f(x) \) with \( x^* = \text{arg min } f(x) \in [a, b] \). Suppose that you want to use a search algorithm that re-uses points at consecutive iterations, such as the Fibonacci or Golden section search methods.

(a) Given two points \( a < p < q < b \) such that \( f(p) < f(q) \), explain why the new point for the next iteration (which narrows the search over \([a, q]\) for the min) should satisfy \( a < r < p \).

(b) Explain why the number of iterations \( n \) for the Fibonacci search method to satisfy a tolerance error \( \epsilon \) is determined by the equation \( (b - a) < 2\epsilon F_n \).

**Problem 2.** Consider the problem of estimating the minimum of the function \( f(x) = \frac{\cos(x)}{x} \) over the interval \( x \in [1, 5] \). You can try `ezplot('cos(x)/x', 1, 5)` in the matlab workspace or define an m-file and use `fplot`.

(a) Plot the function so that you can visualise the problem.

(b) Find an upper bound \( b \) on the minimum using the “improved method” discussed in lectures, using a search parameter value of \( T = 0.15 \). (You can do this writing a matlab code or by hand).

(c) Apply the methods of Fibonacci search, golden section search and Newton-Raphson to within a tolerance error of \( \epsilon = 0.001 \) and record both the estimated minimum \( x^* \) as well as the estimated minimal cost \( f(x^*) \). Compare the methods using three criteria: accuracy, execution time and programming simplicity.

**Problem 3.** Consider the function

\[
f(x) = \begin{cases} 
  x^2 + 2 & \text{if } x \leq 2 \\
  8 - x & \text{otherwise}
\end{cases}
\]

Discuss the applicability of Newton-Raphson and golden section search: will they successfully approximate the minimum in \([-3, 5]\) to within a pre-specified tolerance? [Hint: plot the function].

**Problem 4.** A new model of an office chair can be sold for $70.00 to retail outlets, and for $150.00 to residential customers. However, following a new rebate policy, retail outlets will have a discount of \( 4x_1 \) dollars per chair, when they buy \( x_1 \) chairs, and residential customers will have a discount of \( 15x_2 \) dollars per chair, when they buy \( x_2 \) chairs. The cost of production of chairs per day includes a set up (fixed) cost of $100.00 plus $15.00 per chair. You are the head of production management in the factory. How many chairs should be produced to maximise the total profit per day?

(a) Model the profit function \( f(x_1, x_2) \) as a non-linear function.
(b) Find the critical points.

(c) Determine the maximum.