

620-361 Operations Research Techniques and Algorithms

Assignment 3

Due date: Monday, 26th of May

1. Consider the non-linear program

$$\begin{array}{ll} \min & -x_1 - x_2 \\ \text{s.t.} & x_1 \geq 0 \\ & x_2 \geq 0 \\ & x_1^2 + \frac{1}{2}x_2^2 \leq 1. \end{array}$$

- (a) Write down the Lagrangian of this program.
 - (b) Write down the KKT conditions for this program.
 - (c) Solve the KKT conditions.
 - (d) Show that a constraint qualification holds at your KKT point.
 - (e) Apply a sufficient condition to confirm that your KKT point is a minimum.
2. Consider the non-linear program in question 1.
- (a) Write down the l_2 penalty function for this program with penalty parameter α .
 - (b) Find the gradient of the penalty function when $\alpha = \frac{1}{4}$.
 - (c) Find a minimum of the penalty function when $\alpha = \frac{1}{4}$ and $x_1 > 0$, $x_2 > 0$, and $x_1^2 + \frac{1}{2}x_2^2 > 1$.
 - (d) The above penalty function has a minimum at an infeasible point. Do you expect the penalty function to have an infeasible minimum for $\alpha = 1000$? Why or why not?
3. Show that any point on a line drawn between any two feasible points in a convex program is itself feasible. (In other words, that the feasible region is a convex set).