620-361 Operations Research Techniques and Algorithms

Practice Class 1

1 (a) The Fibonacci numbers F_n satisfy the recursion

$$F_n = F_{n-1} + F_{n-2} \tag{1}$$

with $F_0 = F_1 = 1$.

Show that[1]

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^{n+1} - \left(\frac{1-\sqrt{5}}{2} \right)^{n+1} \right]. \tag{2}$$

(b) Now let $\gamma_n = F_{n-1}/F_n$. Use (1) to derive a quadratic equation for $\gamma \equiv \lim_{n\to\infty} \gamma_n$. (You may assume that this limit exists.)

(c) Solve the equation in (b) and verify that there is a solution in $[0,1]$ that coincides with $\lim_{n\to\infty} F_{n-1}/F_n$ derived from (2).

(d) Apply the golden section search to the function

$$f(x) = (x-\gamma)(x-2\gamma) \tag{3}$$

on the interval $[0,2]$ to obtain an interval of length 2γ that contains the minimum of f.

(e) Apply two iterations of Newton's Method to find the minimum of the function f defined in (3), starting at $x_0 = \gamma$.

(f) Explain the behaviour that you observed in part (e).

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[1] In discrete mathematics, the characteristic equation is used when solving recurrence problems. One can specify a recurrence relation of the form

$$t_n = At_{n-1} + Bt_{n-2},$$

where the value of t_n is dependent on the values of $t_{n?1}$ and $t_{n?2}$. When solving a recurrence relation, the goal is to eliminate this dependency and derive an equation of the form

$$t_n = c_1 {r_1}^n + c_2 {r_2}^n,$$

where c_1 and c_2 are constants and r_1 and r_2 are the roots of the characteristic equation

$$r^2 - Ar - B = 0,$$

where A and B are the constants defined in the original recurrence relation. Source: `http://en.wikipedia.org/wiki/Characteristic_equation`