# 620-361 Operations Research Techniques and Algorithms

## Practice Class 1 Solutions

1. (a) This is given in the course notes but we elaborate more here. Because of the form of the Fibonacci recurrence, it makes sense to try a series of the form $F_n = \gamma^n$ (compare, for example, what would happen if the recurrence was $F_n = 2F_{n-1}$). We plug this into the recurrence, getting

$$\gamma^n = \gamma^{n-1} + \gamma^{n-2}.$$

$$\gamma^2 = \gamma + 1.$$

Using the quadratic formula, this gives

$$\gamma = \frac{\sqrt{5} \pm 1}{2}.$$

Now for both of these $\gamma$'s (which we now call $\gamma_1$ and $\gamma_2$), the sequence $\gamma^n$ satisfies the Fibonacci recurrence. But they do not satisfy the initial conditions $F_0 = F_1 = 1$. To find a sequence which does, we take a linear combination of the two sequences:

$$F_n = A\gamma_1^n + B\gamma_2^n = A\left(\frac{\sqrt{5}+1}{2}\right)^n + B\left(\frac{\sqrt{5}-1}{2}\right)^n.$$

Substituting the initial conditions gives us

$$A + B = 1, \frac{\sqrt{5}+1}{2}A + \frac{\sqrt{5}-1}{2}B = 1.$$

It is easy to solve this linear system to produce the formula given for the Fibonacci numbers.

(b) See notes.

(c) See notes.

(d) We calculate the function at the points $p = 2-2\gamma, q = 2\gamma$. This gives

$$f(p) = (2-3\gamma)(2-4\gamma) = -0.0689, f(q) = (\gamma)(0) = 0.$$

Since $f(p) < f(q)$, the minimum must be less than $q$. So an interval of length $2\gamma$ which contains the minimum of $f$ is $[0, 2\gamma]$.

(e)

$$f(x) = x^2 - 3\gamma x + 2\gamma^2$$

$$f'(x) = 2x - 3\gamma$$

$$f''(x) = 2$$

If $x_0 = \gamma$, then

$$f'(x_0) = -\gamma, f''(x_0) = 2.$$

We then set

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} = \gamma - \frac{-\gamma}{2} = \frac{3\gamma}{2}.$$

This now gives us

$$f'(x_1) = 0, f''(x_1) = 2.$$

As $f'(x_1) = 0$, we have found the minimum and can stop. But nevertheless we plough on gamely!

$$x_2 = x_1 - \frac{f'(x_1)}{f''(x_1)} = \frac{3\gamma}{2} - \frac{0}{2} = \frac{3\gamma}{2} = x_1.$$

(f) As observed above, $x_1$ is the actual minimum of the function, so applying Newton's method to that iterate does not change it.