620-361 Operations Research Techniques and Algorithms

Practice Class 3

1. Let $f : \Re^n \to \Re, f \in C^2$. Consider a stationary point $x^* \in \Re^n$ and let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of $\nabla^2 f(x^*)$. Suppose that $\lambda_1 > 0$ and $\lambda_2 < 0$. Show that x^* is neither a local minimum nor a local maximum.

2. For each of the following matrices, state whether it is positive definite, negative definite, or neither:

   (a)
$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

   (b)
$$\begin{pmatrix} 7 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

   (c)
$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

   (d)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. (a) Show that the rate of convergence of the sequence $x^k = \frac{2k}{4^k + k^4 + 1} \to 0$ is linear.

   (b) What is the rate of convergence of the sequence $x^k = \frac{2x^2 - 3x + 8}{2x^2 + 7x - 2} \to 1$?