620-361 Operations Research Techniques and Algorithms

Practice Class 6

Consider the constrained nonlinear program

$$\min_{x \in \Re^2} f(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 - x_1 + x_2$$
$$\text{subject to} \quad x_1, x_2 \leq 0.$$

1. You can see that this is a 2-D nonlinear optimisation problem with inequality constraints. You would like to solve it using the KKT conditions.
   (a) Write down the Lagrangian and corresponding KKT conditions.
   (b) Solve the system of equations.
2. Now suppose that you were unable to find the solution using the KKT conditions. You resort to a penalty method.
   (a) Write down the l_2-penalty function $P_k(x)$ with penalty parameter k, and explain in general terms, how the ℓ_2-penalty method approximates a solution to a constrained nonlinear program.
   (b) Simplify $P_k(x)$ when $x_1 > 0$ and $x_2 < 0$. Write down $\nabla P_k(x)$ when $x_1 > 0$ and $x_2 < 0$.
   (c) Find a stationary point $x^k = (x_1^k, x_2^k)$ for $P_k(x)$ such that $x_1^k > 0$ and $x_2^k < 0$. Write down the limit $x^* = \lim_{k \to \infty} x^k$.
   (d) Write down an estimate λ^k of the optimal multiplier vector, and find the limit $\lambda^* = \lim_{k \to \infty} \lambda^k$.