

620-361 Operations Research Techniques and Algorithms

Practice Class 7

1. Consider the nonlinear program (NLP):

$$\begin{array}{ll} \min_{x \in \mathfrak{R}^n} & f(x) \\ \text{s.t.} & g_i(x) \leq 0, \quad \forall i = 1, \dots, m. \end{array}$$

Prove that if (NLP) is a convex program with KKT point (x^*, λ^*) , then x^* minimizes the Lagrangian function $L(x, \lambda^*)$, over all $x \in \mathfrak{R}^n$, i.e.,

$$L(x^*, \lambda^*) \leq L(x, \lambda^*)$$

for all $x \in \mathfrak{R}^n$. [Hint: Observe that $L(x, \lambda^*)$ is a function of x , show that x^* is a stationary point of this function, and explain briefly why the function is convex.]

2. Consider the program (LP):

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & Ax = b \\ & x \geq 0 \end{array}$$

where $c \in \mathfrak{R}^n$, $A \in \mathfrak{R}^{n \times m}$ and $b \in \mathfrak{R}^m$.

- (a) Explain briefly why (LP) is a convex program. Write down the Lagrangian function for (LP), using the equality constraint in the form $h(x) = Ax - b = 0$.
- (b) Explain why the Lagrangian dual objective function for (LP) can be given by

$$\psi(\lambda, \eta) = \begin{cases} \eta^T b & \text{if } c^T - \lambda^T - \eta^T A = 0 \\ -\infty & \text{otherwise.} \end{cases}$$

- (c) Write down the Lagrangian dual problem for (LP), and simplify if possible.
- (d) Given $(\hat{\lambda}, \hat{\eta})$ with $\hat{\lambda} \geq 0$ and $c^T - \hat{\lambda}^T - \hat{\eta}^T A = 0$, what can you deduce about the optimal value of (LP)?