

Operations Research Techniques and Algorithms (620-361)

Dr Yao-ban Chan

y.chan@ms.unimelb.edu.au

Telephone: 8344 9073

Office: Room 198, Richard Berry Building

Christina Burt

c.burt@ms.unimelb.edu.au

Telephone: 8344 1797

Office: 139 Barry St

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Today's Lecture

Introduction

Nonlinear programming

Unimodal 1-D unconstrained optimisation

What is Operations Research?

Operations Research (OR) and the Management Sciences (MS) are the professional disciplines that deal with the application of information technology for informed decision-making.

- *Institute for Operations Research and Management Sciences, www.informs.org*

A scientific approach to decision making, which seeks to determine how best to design and operate a system, usually under conditions requiring the allocation of scarce resources

- *from the textbook Operations Research, Winston*

Mathematics of Operations Research

- ▶ linear programming
- ▶ non-linear programming (covered in 361)
- ▶ integer programming
- ▶ decision-making under uncertainty (random processes, statistics, queueing theory, simulation)

Operations Research at Melbourne University

- ▶ The University of Melbourne Operations Research Group (MORe)
- ▶ Australian Research Council Centre of Excellence for Mathematics and Statistics of Complex Systems

Some projects:

- ▶ Communication network design and control
- ▶ Open pit mine optimisation
- ▶ Modelling of patient movements in hospitals
- ▶ Mathematical models of complex queues
- ▶ Transportation and logistics

Communication network design and control

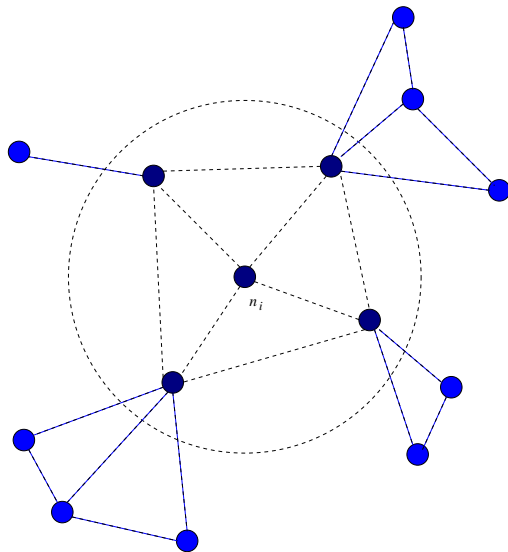


Figure: The roles of a node in the network

Communication network design and control

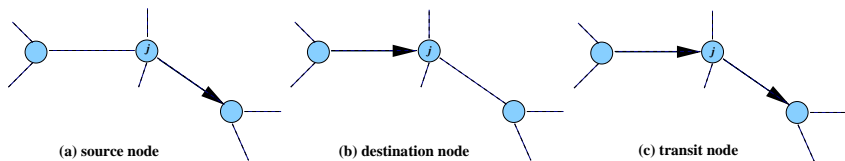


Figure: The roles of a node in the network.

The *SYSTEM*($\mathbf{U}, \mathcal{R}, \mathbf{C}, \mathbf{P}, \mathbf{D}$) can be expressed as:

$$\min - \sum_r U_r(y_r) \quad (1)$$

subject to

$$y_r \geq 0 \quad \forall r \in \mathcal{R}. \quad (2)$$

Solve using the Lagrangian method and Karush-Kuhn-Tucker conditions

Open pit mine optimisation

We are interested in optimising:

- ▶ the shape of the pit;
- ▶ the mining schedule;
- ▶ the selection of equipment; and
- ▶ the allocation of equipment.

Open pit mine optimisation

$$\min f(\mathbf{x}) \quad (3)$$

subject to

$$\mathbf{x} \geq 0 \quad (4)$$

Solve using the Lagrangian method and Karush-Kuhn-Tucker conditions

This course...

- ▶ Website: <http://www.ms.unimelb.edu.au/~s620361>
- ▶ Forum: <http://www.ms.unimelb.edu.au/wiki>
- ▶ Most of the course will cover the topic of non-linear optimisation
- ▶ Focus on algorithms and applications

Class times

- ▶ 3:15-4:15pm Monday, Old Geology Theatre 2
- ▶ 3:15-4:15pm Wednesday, Laby Theatre
- ▶ 3:15-4:15pm Friday, Hercus Theatre
- ▶ Practice class 2:15-3:15pm Friday, Russell Love
- ▶ Computer 'lab' 1-2pm Thursday, Nanson Lab

Important dates

- ▶ Week 1: Select an SSLC rep
- ▶ Week 2: Select projects
- ▶ Week after week 3 but before week 4: Easter
- ▶ Week 4: Assignment 1, project updates
- ▶ Week 8: Assignment 2
- ▶ Week 12: Assignment 3, project talks

Optimisation

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General description of an optimisation problem

- ▶ a constraint set X containing the available “decisions”
- ▶ a cost function $f : X \rightarrow \mathbb{R}$
- ▶ $f(x)$ for some $x \in X$ is a scalar measure of the “undesirability” of choosing decision x
- ▶ the aim is to find an $x^* \in X$ such that

$$f(x^*) \leq f(x), \quad \text{for all } x \in X$$

This course: continuous optimisation

- ▶ each $x \in X$ is an n -dimensional vector, so $X \subseteq \mathbb{R}^n$
- ▶ if $X = \mathbb{R}^n$, then we have a *unconstrained optimisation problem*
- ▶ if X is a subset of \mathbb{R}^n , then we have a *constrained optimisation problem*

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Nonlinear optimisation definition

Nonlinear optimisation, also known as nonlinear programming, describes an optimisation problem in which either

- ▶ the cost function f is non-linear, or
- ▶ the constraint set X is specified by non-linear equations and inequalities

or both!

This is the type of problem which we will focus on in this course...

Example

Consider a production process with two inputs x and y . If production of the item is given by

$$P(x, y) = x^{2/3}y^{1/3}$$

the inputs x and y may be constrained by their total cost

$$p_1x + p_2y = c$$

where p_1, p_2 is the cost of one unit of x, y respectively. Assuming the producer can sell all items, the producer wishes to maximize production P subject to the constraint where c is the total budget allowed for this production process.

Lecture plan

1. Unconstrained optimisation
 - ▶ minimising functions of a single variable
 - ▶ minimising functions of n variables
2. Constrained optimisation
 - ▶ minimising functions of n variables subject to equality and inequality constraints
3. Selected applications

Inventory control

Description given in Operations Research, Winston: To meet demand on time companies often keep on hand stock that is awaiting sale. The purpose of inventory theory is to determine rules that management can use to minimise the costs associated with maintaining inventory and meeting customer demand.

Inventory models aim to answer the following questions:

1. when should an order be placed for a product ?
2. how large should each order be ?

A Problem to Think About

- ▶ Given a continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined on an interval $[a, b]$, find the point $x^* \in [a, b]$ such that $f(x^*) \leq f(x)$ for all $x \in [a, b]$.

Such a point x^* is called a *global minimum* of f on $[a, b]$.

Global and local minima

A *global minimum* x^* of f on $[a, b]$ is a point for which $f(x^*) \leq f(x)$ for all $x \in [a, b]$.

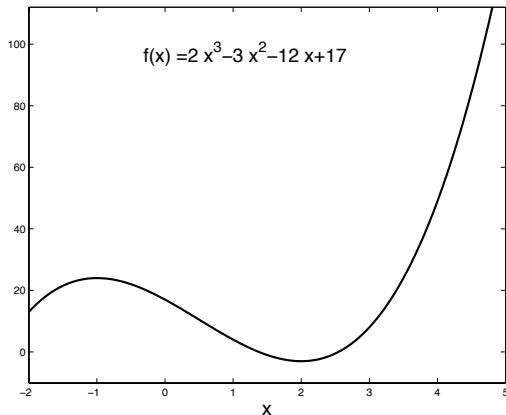
A *local minimum* x^* of f on $[a, b]$ is a point such that there exists a value $\epsilon > 0$ with $f(x^*) \leq f(x)$ for all $x \in [a, b]$ such that $|x - x^*| < \epsilon$.

By the above definitions, a global minimum is also a local minimum.

A local minimum may or may not be a global minimum.

Example:

$$\min_{x \geq -2} f(x) = 2x^3 - 3x^2 - 12x + 17$$



Analytical solution...

If we have a formula for $f(x)$ then we can solve the problem using elementary methods of calculus, that is by setting the first derivative of f equal to zero to determine the stationary points of f , working out which of these are minima, and also checking the value of f at the endpoints.

However, we do not want to make the assumptions that we have a formula for f or, even if we have, that we can necessarily calculate a formula for the derivative. Rather, we want to consider f as a *black box* into which we feed in a value and get another value.

It might actually take a reasonable length of time for the black box to complete the task, so we really want a method that uses the f black box as little as possible.