Operations Research Techniques and Algorithms (620-361)

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Today's Lecture

Comparison of 1D methods

We will go step-by-step through a worked example that illustrates all the methods we have looked at so far.

$$\text{Minimise } f(x) = x^3 - 5x^2 - 3x + 7$$

$$\text{on the interval } x \in [2, 6]$$

Solving analytically:

$$f'(x) = 3x^2 - 10x - 3$$

When this is 0,

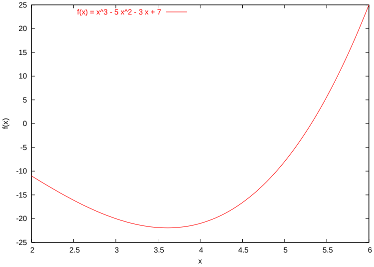
$$x = \frac{5}{3} \pm \frac{\sqrt{34}}{3} = 3.610, -0.27$$

We want the first one.

$$f''(x) = 6x - 10$$

$$f''(3.610) = 11.66 < 0$$

so it is a minimum.



We want to find the minimum to a tolerance of 0.1 for the section searches and with an absolute derivative of less than 0.1 for the derivative methods.

Fibonacci search

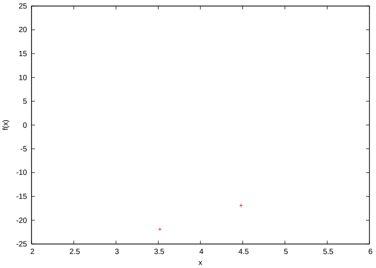
$$\frac{6-2}{F_7} = \frac{6-2}{21} = 0.19 < 0.2$$

so we need 7 calculations.

$$a = 2, b = 6$$

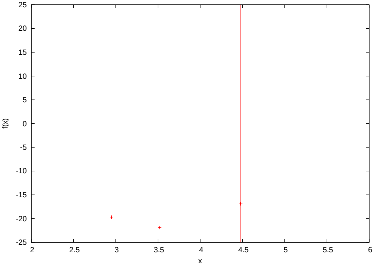
$$p = 6 - \frac{13}{21}(6-2) = 3.52$$

$$q = 2 + \frac{13}{21}(6-2) = 4.48$$



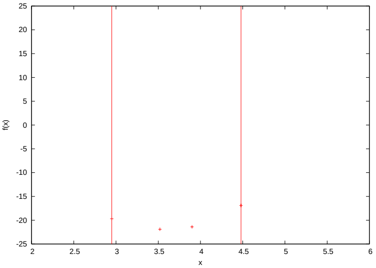
$$f(p) = -21.9 < f(q) = -16.9$$

$$p = 4.48 - \frac{8}{13}(4.48 - 2) = 2.95, q = 3.52$$



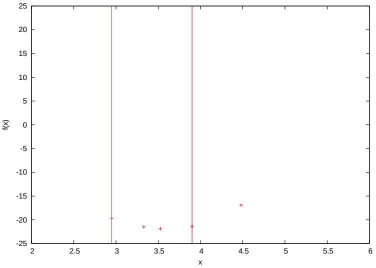
$$f(p) = -19.7 > f(q) = -21.9$$

$$p = 3.52, q = 2.95 + \frac{5}{8}(4.48 - 2.95) = 3.90$$



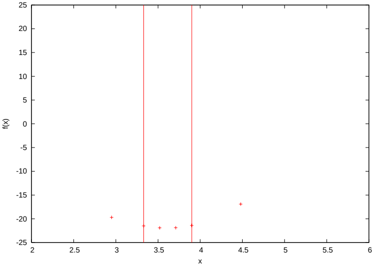
$$f(p) = -21.9 < f(q) = -21.4$$

$$p = 3.90 - \frac{3}{5}(3.90 - 2.95) = 3.33, q = 3.52$$



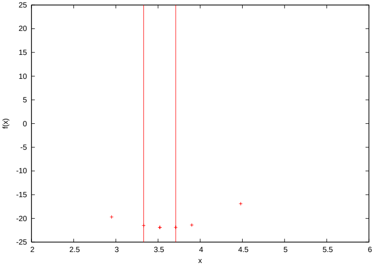
$$f(p) = -21.5 > f(q) = -21.9$$

$$p = 3.52, q = 3.33 + \frac{2}{3}(3.90 - 3.33) = 3.71$$

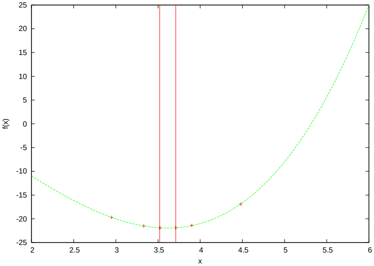


$$f(p) = -21.9 < f(q) = -21.88$$

$$p = 3.52, q = 3.5238$$



$$f(p) = -21.89779 > f(q) = -21.90163$$



Final interval is $[3.52, 3.71]$.Final estimate is 3.617 ± 0.097.

Golden section search

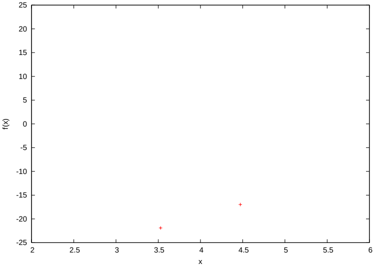
$$\gamma^7(6-2) = 0.13 < 0.2$$

so we need 8 calculations.

$$a = 2, b = 6$$

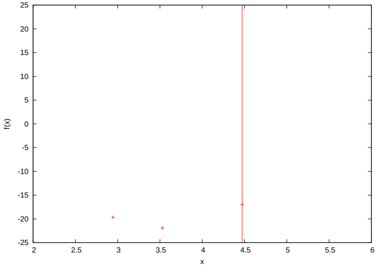
$$p = 6 - \gamma(6-2) = 3.53$$

$$q = 2 + \gamma(6-2) = 4.47$$



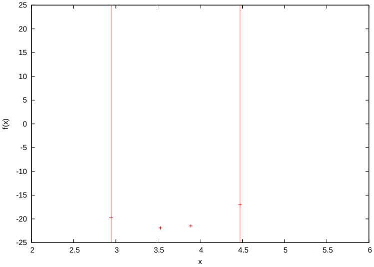
$$f(p) = -21.9055 < f(q) = -16.9737$$

$$p = 4.47214 - \gamma(4.47214 - 2) = 2.94427, q = 3.52786$$



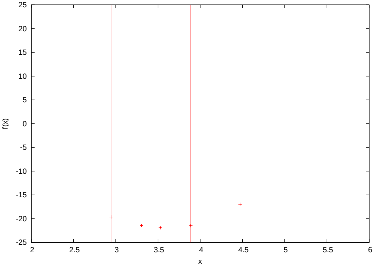
$$f(p) = -19.6534 > f(q) = -21.9055$$

$$p = 3.52786, q = 2.94427 + \gamma(4.47214 - 2.94427) = 3.88854$$



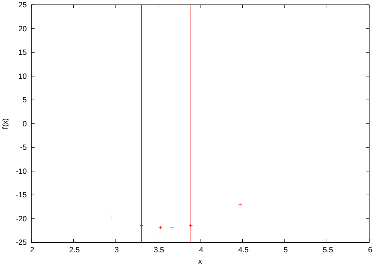
$$f(p) = -21.9055 < f(q) = -21.4717$$

$$p = 3.88854 - \gamma(3.88854 - 2.94427) = 3.30495, q = 3.52786$$



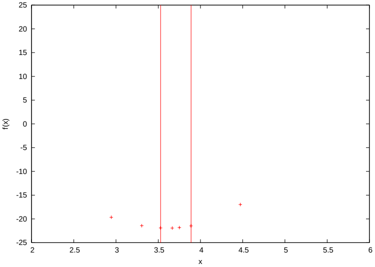
$$f(p) = -21.4294 > f(q) = -21.9055$$

$$p = 3.52786, q = 3.30495 + \gamma(3.88854 - 3.30495) = 3.66563$$



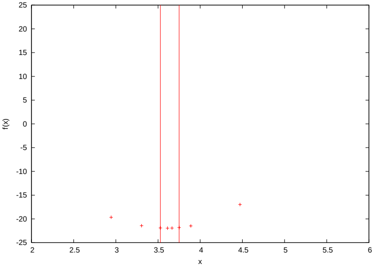
$$f(p) = -21.9055 > f(q) = -21.9266$$

$$p = 3.66563, q = 3.52786 + \gamma(3.88854 - 3.52786) = 3.75078$$

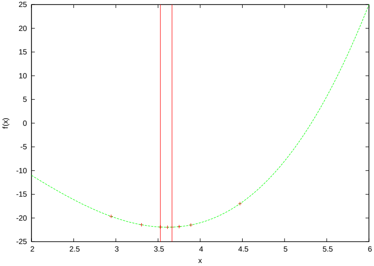


$$f(p) = -21.9266 < f(q) = -21.8268$$

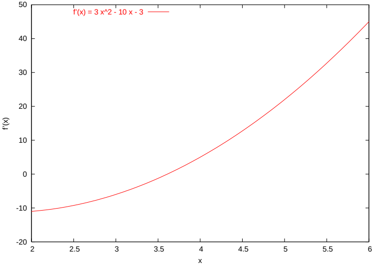
$$p = 3.75078 - \gamma(3.75078 - 3.52786) = 3.61301, q = 3.66563$$



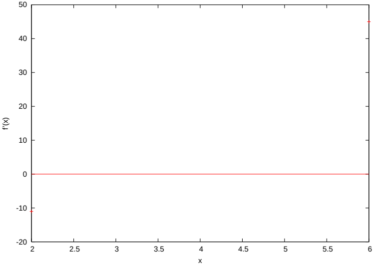
$$f(p) = -21.9446 < f(q) = -21.9266$$



Final interval is $[3.52786, 3.66563]$.Final estimate is 3.597 ± 0.069.

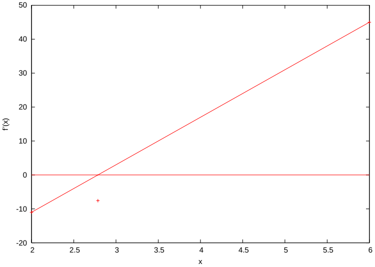

f'(x) = 3 x^2 - 10 x - 3
50
40
30
20
10
0
-10
-20
f'(x)
2
2.5
3
3.5
4
4.5
5
5.5
6
x

False position method



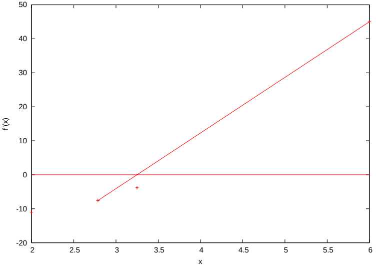
$$a = 2, b = 6$$

$$f'(a) = -11, f'(b) = 45$$



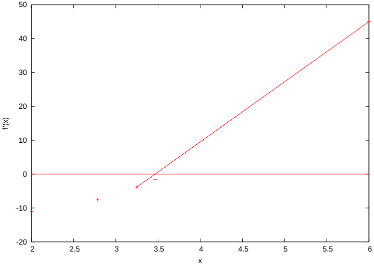
$$p = 2.78571$$

$$f'(p) = -7.57653$$



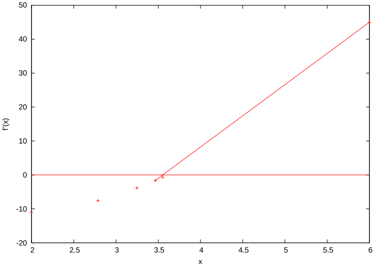
$$a = 2.78571, p = 3.24891$$

$$f'(p) = -3.82287$$



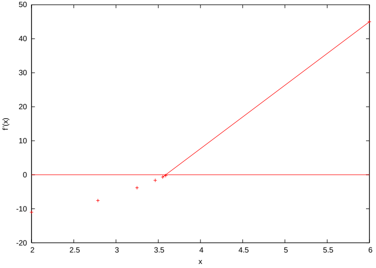
$$a = 3.24891, p = 3.46432$$

$$f'(p) = -1.63865$$



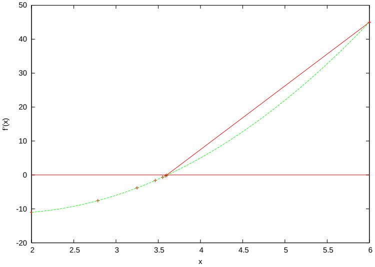
$$a = 3.46432, p = 3.55341$$

$$f'(p) = -0.653909$$



$$a = 3.55341, p = 3.58845$$

$$f'(p) = -0.253522$$

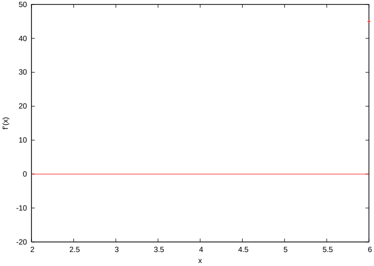


$$a = 3.58845, p = 3.60197$$

$$f'(p) = -0.0971933$$

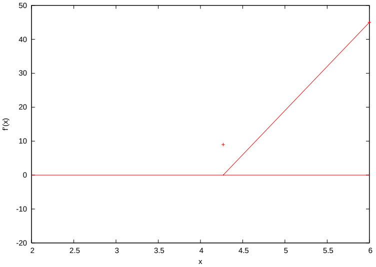
Final interval is $[3.60197, 6]$. Final estimate is 3.602. We needed 8 calculations.

Newton's method



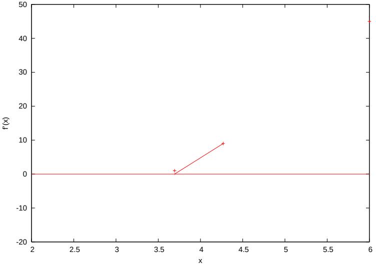
$$a = 6$$

$$f'(a) = 45, f''(a) = 26$$



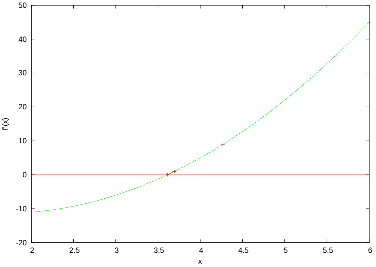
$$p = 4.26923$$

$$f'(p) = 8.98669, f''(p) = 15.6154$$



$$a = p, p = 3.69373$$

$$f'(p) = 0.993608, f''(p) = 12.1624$$



$$a = p, p = 3.61203$$

$$f'(p) = 0.0200223$$

Final estimate is 3.612. We needed 4 calculations, plus 3 f'' calculations.

Comparison

Method	*Solution*	*Calculations*
Exact	3.610	1 long one
Fibonacci	3.617	7
Golden	3.597	8
False position	3.602	8
Newton's	3.612	7

This shows that:

- Fibonacci takes a long time to do! But it is efficient.
- Golden section is less efficient than Fibonacci, but not by a huge amount.
- Newton's method converges very quickly, but requires the most information.