Operations Research Techniques and Algorithms (620-361)

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Outline

Constrained Optimization

We assume throughout this chapter that $f : \Re^n \rightarrow \Re$ is a C^1 objective function.
For $p, q \in \{0, 1, 2, \ldots\}$, let

$$g_1, \ldots, g_p,\ h_1, \ldots, h_q$$

be C^1 functions from $\Re^n$ to $\Re$ and

$$g(x) = (g_i(x))_{i=1}^{p} \text{ and } h(x) = (h_j(x))_{j=1}^{q},$$

so $g : \Re^n \rightarrow \Re^p$ and $h : \Re^n \rightarrow \Re^q$ are C^1 vector-valued functions.

The problem of interest is the *nonlinear program*,

$$\begin{array}{ll} \min\limits_{x} & f(x) \\ \text{subject to} & g(x) \leq 0, \quad h(x) = 0, \end{array} \tag{NLP}$$

where the vector inequality $g(x) \leq 0 \in \Re^p$ means $g_i(x) \leq 0$ for each $i = 1, \ldots, p$, and the vector equality $h(x) = 0$ means $g_j(x) = 0$ for each $j = 1, \ldots, q$.

If $p = 0$, (NLP) is an equality-constrained problem.
If $p = q = 0$, (NLP) is unconstrained and the first-order necessary condition for x^* to minimize f would be that $\nabla f(x^*) = 0$.
We will generalize this stationarity condition to obtain first-order necessary conditions for (NLP). First we review the easier case of nonlinear programs with only equality constraints.

Optimality conditions for equality-constrained optimization

The equality-constrained NLP is

$$\begin{array}{ll} \min_x & f(x) \\ \text{subject to} & h(x) = 0. \end{array} \tag{1}$$

This is just (NLP) with no inequality constraints, $p = 0$. Lagrange gave the first-order necessary conditions for this problem.

The *Lagrangian* function for (1) is

$$L(x,\eta) \ := \ f(x) + \sum_{j=1}^{q} \eta_j h_j(x) \ = \ f(x) + \langle \eta, h(x) \rangle,$$

where $\eta \in \Re^q$. The vector η is called the *Lagrange multiplier* corresponding to $h(x)$, in fact each component η_j of η is a multiplier corresponding to each component $h_j(x)$ of $h(x)$.