620.370 Statistics for Mechanical Engineers — Semester 2, 2009

Homework set 2

Problems to be discussed at next week's tutorial: Quiz 2; 21*, 28, 31, 34.

*21.(b′) Let $X' \stackrel{d}{=} \text{Bi}(100, 0.125)$. Find the mean and variance of X' and a 95% probability interval for X'. How does this relate to 21.(a)?

Homework questions

The first five questions are multiple-choice questions, for each of which exactly one of the proposed alternative answers should be selected.

2.1 If $X \stackrel{d}{=} \text{Bi}(300, 0.25)$, then an approximate 95% probability interval for X is given by

[A.] 300 ± 1; [B.] 75 ± 15; [C.] 100 ± 7.5; [D.] 75 ± 7.5; [E.] 300 ± 112.5

2.2 If $Y \stackrel{d}{=} \text{Hg}(n = 13, R = 13, N = 52)$, then $\Pr(Y \geqslant 5)$ is equal to

[A.] 0.0516; [B.] 0.0802; [C.] 0.1247; [D.] 0.1763; [E.] 0.2060.

Note: (1) use matlab*; (2) this is the probability of obtaining at least five spades in a randonly dealt bridge hand.*

2.3 Consider the following sampling plan. Test a random sample of twenty items chosen from a batch containing a very large (assumed infinite) number of items, and accept the lot if at most one is defective. The value of the operating characteristic (OC) function when the percentage of defectives in the batch is 5% is equal to

[A.] 0.7359; [B.] 0.5179; [C.] 0.3774; [D.] 0.2641; [E.] 0.1887.

Questions 2.4–2.5 refer to the following information:

The Markov chain with state space $\{1, 2\}$ has transition probability matrix given by

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}.$$

2.4 The mean length of a run of 2s is

[A.] 1.1; [B.] 1.5; [C.] 3.3; [D.] 4.2; [E.] 5.0.

2.5 The long run proportion of 2s is

[A.] $\frac{2}{5}$; [B.] $\frac{8}{15}$; [C.] $\frac{3}{5}$; [D.] $\frac{2}{3}$; [E.] $\frac{4}{5}$

2.6 (a) A batch of 50 items contains d defectives. A random sample of 10 items is selected from the batch and each item in the sample is tested. The batch is accepted if there are no defectives in the sample.
Evaluate the operating characteristic function, $P_A(d)$, $d = 0, 1, 2, \ldots, 10$; and sketch the OC curve.

(b) R and S play a game, involving independent points, such that R wins a point with probability 0.55 and S with probability 0.45. The game continues until one or the other has won two more points than the other. *(For example, a tennis game played from deuce.)*
The game can be represented by a Markov chain on $\{-2, -1, 0, 1, 2\}$, where the state is
X = the number of points won by R – the number of points won by S.
Note that if the Markov chain reaches +2, then it stays there and R is the winner; and similarly for –2, in which case S is the winner.
Write down the transition probability matrix for this Markov chain and hence evaluate the probability that R wins the contest.

Quiz 2

Questions 1 and 2 refer to the following information:
A machine produces 100 items per hour in such a way that each item has probability 0.05 of being defective independently of the other items produced.

Q2.1 The probability distribution of the number of non-defective items in an hours production is:

[A.] $\mathrm{Bi}(100, 0.05)$; [B.] $\mathrm{Bi}(100, 0.95)$; [C.] $\mathrm{G}(0.05)$; [D.] $\mathrm{Nb}(100, 0.05)$; [E.] $\mathrm{Nb}(100, 0.95)$.

Q2.2 The probability distribution of the number of defective items produced in producing 100 non-defective items is:

[A.] $\mathrm{Bi}(100, 0.05)$; [B.] $\mathrm{Bi}(100, 0.95)$; [C.] $\mathrm{G}(0.05)$; [D.] $\mathrm{Nb}(100, 0.05)$; [E.] $\mathrm{Nb}(100, 0.95)$.

Q2.3 $\binom{-4}{4}$ is equal to:

[A.] –1; [B.] 4; [C.] –16; [D.] 35; [E.] –256.

Q2.4 Suppose that $X_0, X_1, X_2, \ldots$ are independent identically distributed random variables with pmf:

$$\Pr(X_i = 0) = 0.5,$$
$$\Pr(X_i = 1) = 0.3,$$
$$\Pr(X_i = 2) = 0.2.$$

then $\{X_n, n \geqslant 0\}$ is a Markov chain with transition probability matrix:

[A.] $\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.5 & 0.3 & 0.2 \\ 0.5 & 0.3 & 0.2 \end{bmatrix}$; [B.] $\begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.3 & 0.3 & 0.3 \\ 0.2 & 0.2 & 0.2 \end{bmatrix}$; [C.] $\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$; [D.] $\begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$.

Q2.5 The Markov chain with state space $\{0, 1, 2\}$ has transition probability matrix

$$P = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.4 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}.$$

$\Pr(X_2 = 0 \mid X_0 = 1)$ is equal to

[A.] 0.09; [B.] 0.16; [C.] 0.24; [D.] 0.40; [E.] 0.44.