Answers 2

21.(b′) $\mathrm{E}(X') = 12.5$, $\mathrm{sd}(X') = \sqrt{10.94} = 3.31$.
approximate 95% probability interval = $(7.5 \pm 6.62) = (5.9, 19.1) = (6 \leqslant X' \leqslant 19)$.
$\mathrm{var}(X') > \mathrm{var}(X)$, and so its probability interval is a bit wider than the one for X.
X' relates to sampling with replacement; and X is obtained by sampling without replacement.

Quiz 2

(1) **B** [number of successes in 100 trials with success = non-defective];
(2) **E** [number of failures before the 100th success];
(3) **D** $[= (-4\times - 5\times - 6\times - 7)/(1\times2\times3\times4)]$;
(4) **A** $[\Pr(X_n = j \,|\, X_{n-1} = i) = \Pr(X_n = j)]$;
(5) **E** [from P^2; or LTP: $0.4\times0.6 + 0.4\times0.4 + 0.2\times0.2$].

Homework 2

(2.1) **B** [$\mu \pm 2\sigma$, where $\mu = 300\times\frac{1}{4} = 75$, $\sigma = \sqrt{300\times\frac{1}{4}\times\frac{3}{4}} = 7.5$];
(2.2) **D** [matlab: `1-hygecdf(4,52,13,13)`]; or EXCEL: summing `HYPGEOMDIST(`x`,13,13,52)`; or tedious calculation];
(2.3) **A** [tables or matlab or EXCEL];
(2.4) **E** $[= 1/(1 - 0.8)]$;
(2.5) **C** [several methods: from P^∞; from ratio of mean run lengths; or $0.3/(0.3 + 0.2)$].

(2.6) (a) The number of defectives in the sample, $X \stackrel{\mathrm{d}}{=} \mathrm{Hg}(n{=}10, R{=}d, N{=}50)$; and $P_A(d) = \Pr(X = 0)$. This can be evaluated using MATLAB or EXCEL or a calculator.

d	0	1	2	3	4	5	6	7	8	9	10
$P_A(d)$	1.000	0.800	0.637	0.504	0.397	0.311	0.242	0.187	0.143	0.109	0.083
	1.000	0.668	0.442	0.290	0.189	0.122	0.078	0.049	0.031	0.019	0.012

The last line of the table gives the binomial probabilities (assuming sampling with replacement, or equivalently assuming $N = \infty$), which in this case gives a not very good approximation.

(b) If $X_n = i$ $(i = -1, 0, 1)$, then if R wins, $X_{n+1} = i + 1$; and if S wins $X_{n+1} = i - 1$. The transition probability matrix is given by:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.45 & 0 & 0.55 & 0 & 0 \\ 0 & 0.45 & 0 & 0.55 & 0 \\ 0 & 0 & 0.45 & 0 & 0.55 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Evaluating a large power of P, we obtain

$$P^\infty = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.6705 & 0.0000 & 0.0000 & 0.0000 & 0.3295 \\ 0.4010 & 0.0000 & 0.0000 & 0.0000 & 0.5990 \\ 0.1804 & 0.0000 & 0.0000 & 0.0000 & 0.8196 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

The probability that R wins is therefore 0.5990, since at the start of the game $X = 0$, so we are interested in $p_{02}^{(\infty)}$.

Note: if p denotes the probability that P wins a point, then the probability that R wins the game is $\dfrac{p^2}{p^2 + q^2}$.