

620.370 Statistics for Mechanical Engineers — Semester 2, 2009

Homework set 3

Problems to be discussed at next week's tutorial: Quiz 3; 37, 42, 50, 57.

Homework questions

3.1 Consider a sequence of independent trials each having probability of success p . Let X_n denote the number of successes in n trials; and let Y_r denote the number of trials until the r th success,

Which one of the following is true?

[A.] $\Pr(Y_r > n) = \Pr(X_n < r)$

[B.] $\Pr(Y_r > n) = \Pr(X_n \leq r)$

[C.] $\Pr(Y_r \leq n) = \Pr(X_n > r)$

[D.] $\Pr(Y_r \geq n) = \Pr(X_n < r)$

[E.] $\Pr(Y_r < n) = \Pr(X_n > r)$

3.2 X is a continuous random variable with pdf f and cdf F , and h a small positive number. $\Pr(r - \frac{1}{2}h < X \leq r + \frac{1}{2}h)$ is represented by:

[A.] the slope of the graph $y = F(x)$ at the point $x = r$

[B.] the slope of the graph $y = f(x)$ at the point $x = r$

[C.] the height of the graph $y = f(x)$ at the point $x = r$

[D.] the area under the graph $y = F(x)$ between $x = r - \frac{1}{2}h$ and $x = r + \frac{1}{2}h$.

[E.] the area under the graph $y = f(x)$ between $x = r - \frac{1}{2}h$ and $x = r + \frac{1}{2}h$.

Questions 3.3–3.5 refer to the following information:

The discrete random variable V has pf given by:

v	0	1	2	3	4
$p_V(v)$	0.2	0.3	0.2	0.2	0.1

3.3 The median of V is equal to:

[A.] 1; [B.] 1.3; [C.] 1.5; [D.] 1.7; [E.] 2.

3.4 The mean of V is equal to:

[A.] 1; [B.] 1.3; [C.] 1.5; [D.] 1.7; [E.] 2.

3.5 $E(V^2)$ is equal to:

[A.] 2.25; [B.] 2.89; [C.] 3.20; [D.] 4.00; [E.] 4.50.

3.6 System failures occur in a Poisson process with a rate of 0.4 per week. Let $X(t)$ denote the number of failures in t weeks; let W denote the waiting time until the second failure.

(a) Evaluate $\Pr(X(5) \leq 1)$.

(b) Show that " $W > w$ " = " $X(w) \leq 1$ " and hence obtain an expression for the cdf of W .

(c) Plot the graph of the cdf of W and hence, or otherwise, approximate

- the median and quartiles of W ;
- the mean of W .

(d) Find the pdf of W and sketch its graph. Indicate the median and quartiles, as obtained in (c), on this sketch.

Quiz 3

Q3.1 The Markov chain with state space $\{0, 1, 2\}$ has transition probability matrix P :

$$P = \begin{bmatrix} 0.60 & 0.40 & 0.00 \\ 0.00 & 0.40 & 0.60 \\ 1.00 & 0.00 & 0.00 \end{bmatrix}; \quad P^2 = \begin{bmatrix} 0.36 & 0.40 & 0.24 \\ 0.60 & 0.16 & 0.24 \\ 0.60 & 0.40 & 0.00 \end{bmatrix}; \quad P^4 = \begin{bmatrix} 0.52 & 0.30 & 0.18 \\ 0.46 & 0.36 & 0.18 \\ 0.46 & 0.30 & 0.24 \end{bmatrix}.$$

(a) $\Pr(X_4 = 2 | X_2 = 2)$ is equal to

- A. 0.00
- B. 0.18
- C. 0.24
- D. 0.40
- E. 0.60

(b) On the basis of the above evidence (*no additional calculation is required, though of course you can work it out if you want to waste time*), the long run percentage of zeros is closest to

- A. 20%
- B. 30%
- C. 40%
- D. 50%
- E. 60%

Q3.2 For a Poisson process with rate α , Z_r denotes the time until the occurrence of the r th event, and $X(t)$ denotes the number of events occurring in $(0, t)$. Which one of the following is true?

- A. $\Pr(Z_r \geq t) = \Pr(X(t) \leq r)$
- B. $\Pr(Z_r > t) = \Pr(X(t) < r)$
- C. $\Pr(Z_r < t) = \Pr(X(t) > r)$
- D. $\Pr(Z_r \leq t) = \Pr(X(t) > r)$
- E. $\Pr(Z_r = t) = \Pr(X(t) = r)$

Q3.3 Which one of the following is not a pdf?

- A. $f(x) = 5 \quad (-0.1 < x < 0.1)$
- B. $f(x) = -2x \quad (-1 < x < 0)$
- C. $f(x) = \frac{1}{2}x + \frac{1}{2} \quad (-1 < x < 1)$
- D. $f(x) = 1 \quad (0 < x < 1)$
- E. $f(x) = \frac{2}{3}x \quad (-1 < x < 2)$

Q3.4 Suppose that the graph of the cdf of X , $y = F_X(x)$ near $x = 2$ is as indicated below

Which one of the following is true?

- A. $\Pr(X \geq 2) = 0.4$
- B. $\Pr(X > 2) = 0.4$
- C. $\Pr(X = 2) = 0.4$
- D. $\Pr(X \leq 2) = 0.4$
- E. $\Pr(X \neq 2) = 0.4$

