

## Answers

### Quiz 3

(Q3.1a) A [  $= p_{22}^{(2)}$  ];

(Q3.1b) D [ it looks like  $P^n \rightarrow \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.5 & 0.3 & 0.2 \\ 0.5 & 0.3 & 0.2 \end{bmatrix}$ ; actually  $P^\infty = \begin{bmatrix} 0.4839 & 0.3226 & 0.1935 \\ 0.4839 & 0.3226 & 0.1935 \\ 0.4839 & 0.3226 & 0.1935 \end{bmatrix}$  ];

(Q3.2) B [ if  $Z_r > t$  then the  $r$ th event occurs after time  $t$ , and therefore  $X(t)$  must be less than  $r$ ; if  $X(t) < r$ , then the  $r$ th event must occur after time  $t$ , i.e.  $Z_r > t$  ];

(Q3.3) E [ this function is negative for  $x < 0$  ];

(Q3.4) B [ from the graph,  $\Pr(X < 2) = 0.4$ ,  $\Pr(X = 2) = 0.2$  and  $\Pr(X > 2) = 0.4$  ].

### Homework 3

(3.1) A [if  $Y_r > n$  then the  $r$ th success occurs after the  $n$ th trial, and therefore  $X_n$  must be less than  $r$ ; if  $X_n < r$ , then the  $r$ th success must occur after the  $n$ th trial];

(3.2) E [  $\Pr(a < X \leq b) = F(b) - F(a)$  ];

(3.3) C [  $F(x) = 0.5$  for  $1 \leq x < 2$ , so the median is  $(1 + 2)/2$  ];

(3.4) D [  $= 0 \times 0.2 + 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.2 + 4 \times 0.1$  ];

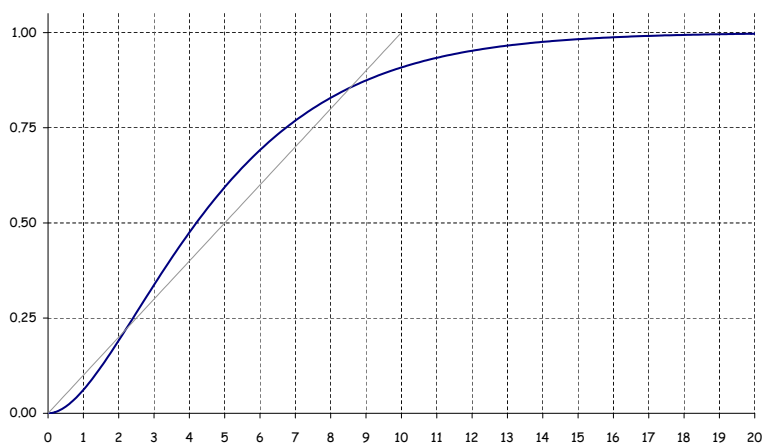
(3.5) E [  $= 0^2 \times 0.2 + 1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.2 + 4^2 \times 0.1$  ].

(3.6) (a)  $X(5) \stackrel{d}{=} \text{Pn}(2)$ , so  $\Pr(X(5) \leq 1) = 0.4060$ .

(b) Since  $X(w) \stackrel{d}{=} \text{Pn}(0.4w)$ ,  $\Pr(X(w) \leq 1) = e^{-0.4w} + 0.4we^{-0.4w}$ .

Therefore,  $F(w) = 1 - (1 + 0.4w)e^{-0.4w}$  ( $w > 0$ )

The graph of  $F$  is plotted below:



From the graph, reading off where  $F = 0.25$ ,  $0.5$  and  $0.75$ , we obtain

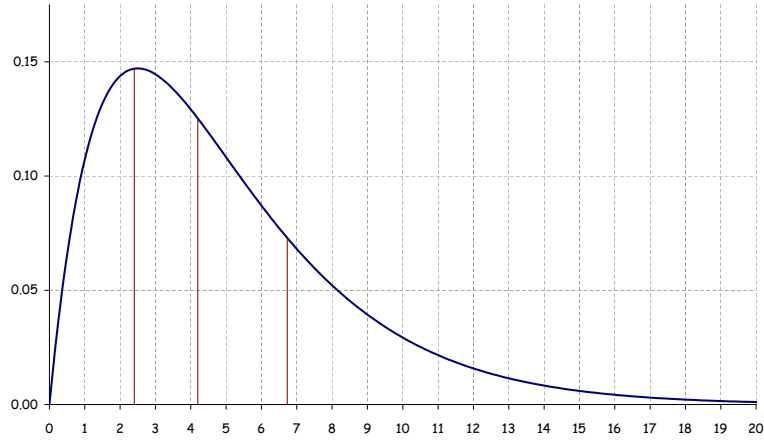
$$c_{0.25} \approx 2.4, c_{0.5} \approx 4.2 \text{ and } c_{0.75} \approx 6.7.$$

Solving numerically gives  $c_{0.25} = 2.403$ ,  $c_{0.5} = 4.196$  and  $c_{0.75} = 6.732$ .

The area above the cdf (and below 1) gives the mean (see notes p2.5). This area can be approximated by drawing a straight line from the origin, so that it looks like the area above the straight line is about the same as the area above the cdf. This line is indicated on the graph. This gives  $\mu \approx 5$ , using area of triangle  $\approx \frac{1}{2} \times 1 \times 10$ .

(c)  $f = F' = (1 + 0.4w)0.4e^{-0.4w} - (0.4)e^{-w}$ , using the product rule; so  
 $f(w) = 0.16we^{-0.4w}$  ( $w > 0$ ).

The graph of  $f$  is plotted below:



The median and quartiles are indicated. These divide the area under the pdf graph into four equal parts: 0.25 in each.