620.370 Statistics for Mechanical Engineers — Semester 2, 2009

Homework set 4

Problems to be discussed at next week's tutorial: Quiz 4; 61, 67, 77, 85.

Homework questions

Questions 4.1 & 4.2 refer to the following information:

Y is a continuous random variable with pdf given by:
$f_Y(y) = 3y^2 \quad (0 < y < 1)$

4.1 $\mathrm{var}(Y)$ is equal to:

[A.] $\frac{3}{2}$; [B.] $\frac{3}{4}$; [C.] $\frac{3}{5}$; [D.] $\frac{3}{20}$; [E.] $\frac{3}{80}$.

4.2 $\mathrm{var}(\frac{1}{Y})$ is equal to:

[A.] $\frac{3}{4}$; [B.] $\frac{4}{3}$; [C.] $\frac{3}{2}$; [D.] 3; [E.] ∞

4.3 If $\mathrm{E}(X) = \mu \quad \mathrm{var}(X) = \sigma^2$, then $\mathrm{var}(X^3)$ is approximately equal to:

[A.] $3\sigma^2$; [B.] $3\mu^2\sigma^2$; [C.] $9\mu^4\sigma^2$; [D.] $9\mu^4\sigma^4$; [E.] σ^6.

4.4 If T has hazard function $h(t) = 3(t-1)^2$, $t \geqslant 0$, then the cdf of T is equal to

[A.] $e^{-(t-1)^3}$, $(t \geqslant 0)$; [B.] $1 - e^{-(t-1)^3}$, $(t \geqslant 0)$; [C.] $e^{1-(t-1)^3}$, $(t \geqslant 0)$;
[D.] $1 - e^{1-(t-1)^3}$, $(t \geqslant 0)$; [E.] $e^{-1+(t-1)^3}$, $(t \geqslant 0)$.

4.5 If $X \stackrel{\mathrm{d}}{=} \mathrm{N}(0,1)$, then $\Pr(X > 2 \,|\, X > 1.5)$ is equal to

[A.] 0.227; [B.] 0.341; [C.] 0.773; [D.] 0.909; [E.] 0.955.

4.6 (a) Let $W = X^2Y$, where X and Y are independent random variables such that $\mathrm{E}(X) = \mathrm{E}(Y) = 10$ and $\mathrm{sd}(X) = \mathrm{sd}(Y) = 1$.

i. Find $\mathrm{E}(W)$.

ii. Find an approximate value for $\mathrm{E}(W^2)$; and hence find an approximate value for $\mathrm{sd}(W)$.

Note: You will need to use the result that if U and V are independent random variables, then $\mathrm{E}(UV) = \mathrm{E}(U)\mathrm{E}(V)$.

(b) Suppose that $X \stackrel{\mathrm{d}}{=} \mathrm{N}(45, 4^2)$.

i. Find the probability that X exceeds the threshold, $t = 40$.

ii. Suppose that instead of a constant threshold, the threshold is actually random: $T \stackrel{\mathrm{d}}{=} \mathrm{N}(40, 3^2)$. Find the probability that X exceeds the random threshold, T.

Quiz 4

Questions Q4.1–Q4.3 refer to the information below:

T is a continuous random variable with pdf given by:

$$f(t) = \tfrac{2}{9}(4-t) \quad (1 < t < 4)$$

Q4.1 The probability $\Pr(T > 2.5)$ is equal to:

[A.] $\frac{2}{3}$; [B.] $\frac{1}{2}$; [C.] $\frac{1}{3}$; [D.] $\frac{1}{4}$; [E.] $\frac{1}{6}$.

Q4.2 The mean of T is:

[A.] 1.5; [B.] 2.0; [C.] 2.5; [D.] 3.0; [E.] 3.5.

Q4.3 The hazard function of the failure time T on $1 < t < 4$ is

[A.] $\dfrac{9}{2(4-t)}$; [B.] $\dfrac{8-2t}{5+4t-t^2}$; [C.] $\frac{8}{9}$; [D.] $\dfrac{8-2t}{1+8t-2t^2}$; [E.] $\dfrac{2}{4-t}$.

Q4.4 X and Y are independent random variables with $\sigma_X = 4$ and $\sigma_Y = 3$.
If $Z = X + Y + 2$, then the standard deviation of Z is

[A.] 5; [B.] 9; [C.] $\sqrt{27}$; [D.] 7; [E.] $\sqrt{29}$.

Q4.5 If $X \stackrel{\mathrm{d}}{=} \mathrm{N}(0, 4)$, then $\Pr(X \geqslant 1)$ is equal to

[A.] 0.1587; [B.] 0.25; [C.] 0.3085; [D.] 0.4013; [E.] 0.4602.