

## Answers

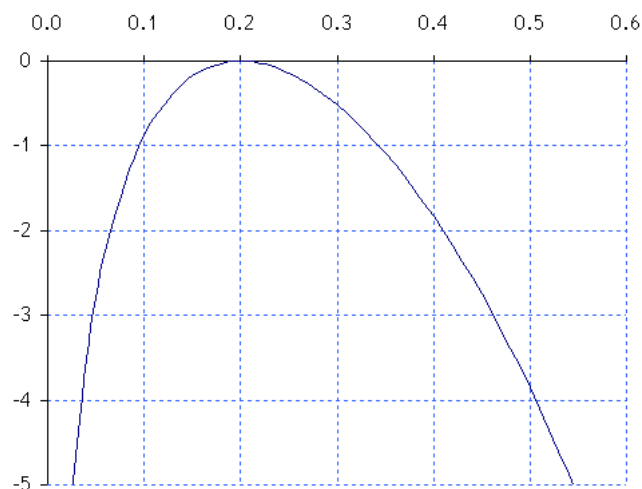
## Quiz 7

- (Q7.1) **B** [ using the diagram in the tables, or MATLAB. ];
- (Q7.2) **A** [  $\hat{p} = 0.1$ ; 95% CI:  $0.1 \pm 1.96\sqrt{\frac{0.1 \times 0.9}{100}} = (0.0412, 0.1588)$ . ];
- (Q7.3) **E** [  $\frac{1.96\sigma}{\sqrt{10}} = 2.48$ ,  $\frac{1.96\sigma}{\sqrt{n}} = 1.24$ ; so  $\sqrt{n} = \frac{1.96\sigma}{1.24} = \frac{2.48\sqrt{10}}{1.24} = 2\sqrt{10}$ . ];
- (Q7.4) **A** [ size =  $\Pr(T \geq 7)$ , where  $T \stackrel{d}{=} \text{Bi}(10, 0.4)$ ; tables  $\Rightarrow$  size = 0.0548.  
power =  $\Pr(T \geq 7)$ , where  $T \stackrel{d}{=} \text{Bi}(10, 0.8)$ ; tables  $\Rightarrow$  power = 0.8791.];
- (Q7.5) **B** [  $P \leq \alpha \Rightarrow$  reject  $H_0$  at level  $\alpha$ . We can't conclude anything about the probability of the truth of the null hypothesis from these data.].

## Homework 7

- (7.1) **E** [  $2.40 \pm 2.131 \times \frac{1.2}{\sqrt{16}}$  ];
- (7.2) **C** [  $\frac{15 \times 1.2^2}{(6.262, 27.49)}$  ];
- (7.3) **A** [  $2.40 \pm 2.131 \times 1.2 \times \sqrt{1 + \frac{1}{16}}$  ];
- (7.4) **D** [  $P = 2 \Pr(Z < -0.81) = 2 \times 0.209 = 0.418$  ];
- (7.5) **C** [  $4S^2 \stackrel{d}{=} \chi_4^2 \Rightarrow \Pr(4S^2 > 9.488) = 0.05 \Rightarrow \Pr(S^2 > 2.372) = 0.05$ . ].

- (7.6) (a) i.  $\frac{\partial \ln L}{\partial \theta} = \frac{4}{\theta} - \frac{16}{1-\theta}$ .  $\frac{\partial \ln L}{\partial \theta} = 0 \Rightarrow \hat{\theta} = 0.2$ .  
 $\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{4}{\theta^2} - \frac{16}{(1-\theta)^2} \Rightarrow \text{se}(\hat{\theta}) = 1 / \sqrt{\frac{4}{0.2^2} + \frac{16}{0.8^2}} = 0.089$ .  
*Note: These are identical to  $\hat{\theta} = \frac{4}{20} = 0.2$ , and  $\text{se}(\hat{\theta}) = \sqrt{\frac{0.2 \times 0.8}{20}} = 0.089$ .*
- ii. The graph of the relative log-likelihood is shown below:



From the graph, or by solving (numerically)  $\text{RLL} = -2$ , we obtain the approximate 95% confidence interval:  $0.065 < p < 0.410$ .

- iii. The diagram in the Tables gives  $0.06 < p < 0.44$ ; MATLAB gives  $0.057 < p < 0.437$ .
- (b) i. reject  $H_0$  since  $0.5 \notin \text{CI}$ .  
 ii.  $P = 2 \Pr(X' \leq 4) = 2 \times 0.0059 = 0.012$ ; since  $P < 0.05$ , we reject  $H_0$ .  
 iii. size =  $\Pr(\text{reject } H_0 | p=0.5) = \Pr(X \leq 5) + \Pr(X \geq 15)$ , where  $X \stackrel{d}{=} \text{Bi}(20, 0.5)$ ;  
 $\therefore$  size =  $0.0207 + 0.0207 = 0.0414$ .  
 power =  $\Pr(\text{reject } H_0 | p=0.2) = \Pr(X \leq 5) + \Pr(X \geq 15)$ , where  $X \stackrel{d}{=} \text{Bi}(20, 0.2)$ ;  
 $\therefore$  power =  $0.8042 + 0.0000 = 0.8042$ .