

620.370 Statistics for Mechanical Engineers — Semester 2, 2009

Homework set 9

Problems to be discussed at next week's tutorial: Quiz 9; 158, 163, 164, 167, 170.

1. Independent random samples were taken from two normal populations with equal variances in order to test the hypothesis $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$. From the samples the following quantities were obtained:

$$n_1 = 6, \bar{x}_1 = 15.2, s_1^2 = 3.0; \quad n_2 = 11, \bar{x}_2 = 12.2, s_2^2 = 4.5.$$

The value of the t-statistic is

- [A.] 2.91; [B.] 2.96; [C.] 3.05; [D.] 3.15; [E.] 3.30.

2. If $U \stackrel{d}{=} \chi_6^2$, which one of the following gives a 95% probability interval for U ?
 [A.] (1.24, 14.45); [B.] (1.64, 12.59); [C.] (-12.83, 12.83); [D.] (0.83, 12.83); [E.] (1.69, 16.01).
3. If $F \stackrel{d}{=} F_{6,12}$, which one of the following gives a 95% probability interval for F ?
 [A.] (0.27, 5.37); [B.] (0.19, 5.37); [C.] (0.25, 3.00); [D.] (0.27, 3.73); [E.] (0.19, 3.73).

Questions 4 and 5 refer to the following information:

In testing the equality of the means of five normal populations, samples of 3, 5, 4, 2 and 3 are obtained on these populations, giving the following sums of squares:

between groups SS, $B = 60$; within groups SS, $W = 12$; total SS, $T = 72$.

4. The value of the F -statistic is such that
 [A.] $0 < F < 4$; [B.] $4 < F < 8$; [C.] $8 < F < 12$; [D.] $12 < F < 16$; [E.] $F > 16$.
5. The F -statistic has degrees of freedom
 [A.] 4, 5; [B.] 12, 5; [C.] 4, 12; [D.] 5, 12; [E.] 4, 13.

6. The following observations are obtained on two treatments:

treatment 1	33.3	37.3	35.1	28.7	25.5	35.8	41.2	33.5	24.0	29.5
treatment 2	35.5	40.0	38.5	29.3	27.5	36.9	43.5	37.3	27.5	30.6

It can be assumed that the observations are independent and normally distributed with equal variances.

Let δ denote the increase in mean by using treatment 2 rather than treatment 1.

- (a) Test for the difference in the effects of the two treatments using an independent samples t-test. Derive a 95% confidence interval for δ .

Now suppose that the columns actually correspond to blocks:

	a	b	c	d	e	f	g	h	i	j
treatment 1	33.3	37.3	35.1	28.7	25.5	35.8	41.2	33.5	24.0	29.5
treatment 2	35.5	40.0	38.5	29.3	27.5	36.9	43.5	37.3	27.5	30.6

- (b) Test for the difference in the effects of the two treatments using a paired-samples t-test. Derive a 95% confidence interval for δ .

Why is this interval narrower than the one derived in (a)?

- (c) Obtain the analysis of variance table in each of the following cases:
- i. one-way ANOVA with groups = treatments, and compare the result with the independent samples analysis of (a);
 - ii. two-way ANOVA with rows = treatments and columns = blocks, and compare the result with the paired samples analysis of (b).

Quiz 9

Q9.1 In order that

$$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \stackrel{d}{=} t_{n_1+n_2-2}$$

where $S^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$, which one of the following conditions is not necessary?

- A. the samples on X_1 and X_2 are independent;
- B. X_1 and X_2 are normally distributed;
- C. $\sigma_1^2 = \sigma_2^2$;
- D. n_1 and n_2 are large;
- E. none of the above.

Questions Q9.2 and Q9.3 refer to the following information:

Independent random samples of nine observations on $X_1 \stackrel{d}{=} N(\mu_1, \sigma_1^2)$ and twenty-five observations on $X_2 \stackrel{d}{=} N(\mu_2, \sigma_2^2)$ are obtained.

These observations give:

$$\begin{aligned} \bar{x}_1 &= 17.32 & s_1^2 &= 2.25 \\ \bar{x}_2 &= 16.24 & s_2^2 &= 2.25 \end{aligned}$$

Q9.2 A 95% confidence interval for σ_1^2/σ_2^2 is

- A. (0.25, 2.76)
- B. (0.32, 2.36)
- C. (0.36, 3.95)
- D. (0.40, 3.64)
- E. (0.42, 4.12)

Q9.3 In testing $H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$, the significance level, P , is such that

- A. $P < 0.01$
- B. $0.01 < P < 0.025$
- C. $0.025 < P < 0.05$
- D. $0.05 < P < 0.10$
- E. $P > 0.10$

Questions Q9.4 and Q9.5 refer to the following information:

Consider the analysis of variance table:

	df	SS	MS	F
between groups	2	*	*	*
within groups	*	5	*	
	12	25		

in which the stars denote missing values.

Q9.4 The value of the F -statistic is

- A. 2
- B. 5
- C. 10
- D. 15
- E. 20

Q9.5 The value of s^2 is

- A. 10
- B. 5
- C. 2
- D. 1
- E. 0.5