620.370 Statistics for Mechanical Engineers — Semester 2, 2009

Answers to Homework 9

Quiz 9 answers:

1. D. [by definition, see ES p.6.5]
2. C. [$\frac{\sigma_1^2}{\sigma_2^2} \sim \frac{s_1^2}{s_2^2}\mathrm{F}_{24,8} \;\Rightarrow\; \frac{\sigma_1^2}{\sigma_2^2} \sim 1 \times (1/2.779, 3.947) = (0.360, 3.947)$]
3. D. [$t = \frac{17.32-16.24}{\sqrt{2.25(\frac{1}{9}+\frac{1}{25})}} = \frac{1.08}{0.583} = 1.852$; $df = 32$;
   $c_{0.95}(\mathrm{t}_{32}) = 1.694 < 1.852 < c_{0.975}(\mathrm{t}_{32}) = 2.037 \;\Rightarrow\; 0.05 < \mathrm{p} < 0.10 \quad (\mathrm{p} = 0.073)$]
4. E. [

between	2	20	10	20
within	10	5	0.5	
total	12	25		

   $\Rightarrow\; F = 20.$]
5. E. [anova table $\;\Rightarrow\; s^2 = \text{within.MS} = 0.5$]

Homework 9 answers:

1. B. [equal variances: $s^2 = \frac{5\times3.0+10\times4.5}{15} = 4.0$; $t = \frac{15.2-12.2}{\sqrt{4.0(\frac{1}{6}+\frac{1}{11})}} = \frac{3.0}{1.015} = 2.96.$]
2. A. [tables give $c_{0.025}(\chi^2_6) = 1.237$ and $c_{0.975}(\chi^2_6) = 14.45$]
3. E. [$c_{0.025}(\mathrm{F}_{6,12}) = 1/c_{0.975}(\mathrm{F}_{12,6}) = 1/5.366 = 0.186$; $c_{0.975}(\mathrm{F}_{6,12}) = 3.738$]
4. D. [$k = 5$, $N = 3+5+4+2+3 = 17$

between	4	60	15	15
within	12	12	1	
total	16	72		

   $\Rightarrow\; F = 15.$]
5. C. [from anova table: $df = 4, 12$]
6. (a) $n_1 = 10 \quad \bar{x}_1 = 32.39 \quad s_1 = 5.40$
   $n_2 = 10 \quad \bar{x}_2 = 34.66 \quad s_2 = 5.60$

   $s^2 = \frac{1}{2}(s_1^2 + s_2^2) \;\Rightarrow\; s = 5.50$; $\mathrm{se}(\bar{x}_1 - \bar{x}_2) = 5.50\sqrt{\frac{1}{10} + \frac{1}{10}} = 2.46.$

   $t = \frac{-2.27}{2.46} = -0.92$, cf. $c_{0.975}(\mathrm{t}_{18}) = 2.101$;
   so we do not reject H_0 (no significant difference between the means using this approach).

   95% CI for $\mu_1 - \mu_2$: $(-2.27 \pm 2.101\times2.46) = (-7.44, 2.90)$.

   (b) paired samples: consider the sample of differences

   $\{2.2, 2.7, 3.4, 0.6, 2.0, 1.1, 2.3, 3.8, 3.5, 1.1\}$.

   for which $n = 10$, $\bar{d} = 2.27$, $s_d = 1.102$

   $t = \frac{2.27}{1.102/\sqrt{10}} = 6.52$, cf. $c_{0.975}(\mathrm{t}_9) = 2.262$;

   so we reject H_0: there is a (highly) significant difference between the treatment means.

   95% CI for $\mu_d = \mu_1 - \mu_2$: $(2.27 \pm 2.262\times\frac{1.102}{\sqrt{10}}) = (1.48, 3.06)$.

   The second approach is (far) more efficient: the variation between blocks is removed.

(c) One-way ANOVA:

Source	DF	SS	MS	F	P
t	1	25.8	25.8	0.85	0.368
Error	18	544.4	30.2	S = 5.50	R-Sq = 4.52%
Total	19	570.2			

Two-way ANOVA:

Source	DF	SS	MS	F	P
t	1	25.765	25.765	42.47	0.000
b	9	538.972	59.886	98.70	0.000
Error	9	5.460	0.607	S = 0.779	R-Sq = 99.0%
Total	19	570.197			

$t_a^2 = (-0.92)^2 = 0.85 = F_1$; $\mathrm{df}_a = \mathrm{df}_1 = 18$; $s_a^2 = 5.50^2 = 30.2 = s_1^2$

$t_b^2 = 6.52^2 = 42.47 = F_2$; $\mathrm{df}_b = \mathrm{df}_2 = 9$; $s_d^2 = 1.214 = 2s_2^2$.