620.370 Statistics for Mechanical Engineers — Semester 2, 2009

Homework set 11

Problems to be discussed at next week's tutorial: Quiz 11; 200, 202, 208, 213.

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1. In a 2^4 factorial experiment, (number of main effects, number of two-factor interactions, number of higher-order interactions) is equal to
   [A.] $(4, 6, 5)$; [B.] $(4, 4, 6)$; [C.] $(4, 6, 6)$; [D.] $(4, 4, 7)$; [E.] $(3, 5, 7)$; [F.] $(3, 6, 6)$.

Problems 11.2–11.3 refer to the following information:
A blocked four-replicate 2^3 experiment on factors U, V and W is run in four blocks, with one complete replicate in each block.

2. Let s denote the square root of the error mean square. The standard error of the estimate of the main effect of U is given by
   [A.] s; [B.] $s/\sqrt{2}$; [C.] $s/\sqrt{4}$; [D.] $s/\sqrt{8}$; [E.] $s/\sqrt{16}$; [F.] $s/\sqrt{32}$.

3. Preliminary analysis indicates that of the factor effects, only U, V, UV and W are important, so a model with only these effects is fitted. The resulting error MS is then used as s^2, the estimate of the error variance. The number of degrees of freedom of s^2 is
   [A.] 31; [B.] 28; [C.] 24; [D.] 21; [E.] 15.

4. Consider a 2^4 factorial experiment in factors J, K, L and M. Assume independent normally distributed errors with variance σ^2. Which one of the following is false?
   [A.] If the effect of L is zero, then $\text{SS}(L) \stackrel{\text{d}}{=} \sigma^2\chi^2_1$.
   [B.] If the F-value for J is greater than the critical value then this indicates that the effect of J is positive.
   [C.] The main effect of K measures the difference in the mean value of the response variable with K at level 1 and with K at level 0.
   [D.] The interaction JM measures the difference in effect of J at the different levels of M.
   [E.] If the p-value for the effect KL is equal to 0.078, we conclude that there is no significant evidence against the hypothesis that $KL = 0$.

5. A bivariate data set yielded the following summary statistics:
   $n = 27$, $\bar{x} = 20$, $\bar{y} = 30$; $\Sigma(x-\bar{x})^2 = 100$, $\Sigma(x-\bar{x})(y-\bar{y}) = 50$, $\Sigma(y-\bar{y})^2 = 125$.
   For these data:
   [A.] $\hat{\beta} = 0.4$, $\text{se}(\hat{\beta}) = 0.1$; [B.] $\hat{\beta} = 0.4$, $\text{se}(\hat{\beta}) = 0.2$; [C.] $\hat{\beta} = 0.4$, $\text{se}(\hat{\beta}) = 0.3$;
   [D.] $\hat{\beta} = 0.5$, $\text{se}(\hat{\beta}) = 0.1$; [E.] $\hat{\beta} = 0.5$, $\text{se}(\hat{\beta}) = 0.2$; [F.] $\hat{\beta} = 0.5$, $\text{se}(\hat{\beta}) = 0.3$;

6. (a) Twelve experimental units are available in three blocks of four. It is required to run an experiment to compare four treatments. Give an appropriate assignment of treatments to experimental units.
   (b) The experiment described in (a) is carried out, with results:

	B_1	B_2	B_3
T_1	46.0	51.6	54.2
T_2	55.2	55.7	63.3
T_3	48.0	54.5	59.3
T_4	61.1	63.1	66.9

$\Rightarrow$

source	df	SS	MS
blocks	**	***	***
treatments	**	***	***
error	**	***	2.972
total	**	445.02	

   i. Using MATLAB [`anovan(y,{t b},1)`], or otherwise, complete the analysis of variance table, and hence test the significance of the treatment effects.

   The experiment above is actually a three replicate 2^2 factorial experiment, with $T_1 = P_0Q_0$, $T_2 = P_1Q_0$, $T_3 = P_0Q_1$ and $T_4 = P_1Q_1$. This allows the treatment sum of squares to be split up into components due to P, Q and PQ.

   ii. Using MATLAB [`anovan(y,{p q},2)`], or otherwise, show that $SS(P) = 222.74$, and complete the split-up of the treatment sum of squares.
   iii. Show that the effect of P is highly significant.
   iv. Give an estimate of the effect of P, and its standard error.

Quiz 11

Q11.1–11.4 refer to the following information:

For a set of bivariate data, the following values are obtained:

$n = 100$	$\sum xy = 2200$	$\sum(x-\bar{x})(y-\bar{y}) = -200$
$\sum x = 400$	$\sum x^2 = 1800$	$\sum(x-\bar{x})^2 = 200$
$\sum y = 600$	$\sum y^2 = 4400$	$\sum(y-\bar{y})^2 = 800$

Q11.1 For the given information, the sample variances and covariances (s_x^2, s_{xy}, s_y^2) are equal to

[A.] $(18.2, 11.2, 44.4)$; [B.] $(1.42, 1.42, 2.84)$; [C.] $(1.42, -1.42, 2.84)$;
[D.] $(2.02, 2.02, 8.08)$; [E.] $(2.02, -2.02, 8.08)$.

Q11.2 The fitted straight line regression of Y on X obtained using the method of least squares is given by

[A.] $y = 10 - x$; [B.] $y = 6 - x$; [C.] $y = 2 + x$; [D.] $y = 7 - 0.25x$; [E.] $y = 5 + 0.25x$.

Q11.3 The estimate of the error variance, s^2, is closest to

[A.] 2; [B.] 4; [C.] 6; [D.] 8; [E.] 10.

Q11.4 The sample correlation coefficient, r, is closest to

[A.] –0.8; [B.] –0.5; [C.] –0.1; [D.] 0.5; [E.] 0.8

Q11.5 The regression of Y on X is

[A.] the tendency of Y to be nearer to X due to the relationship between X and Y.

[B.] the function specifying the mean value of X for a given value of Y.

[C.] the relationship indicating the dependence of Y on X.

[D.] the function specifying the mean value of Y for a given value of X.

[E.] a measure of the extent to which the relationship between X and Y tends to regress as the variables move away from their mean values.