

Answers to Homework 11

Quiz 11 answers:

Q11.1 E. [$(s_x^2, s_{xy}, s_y^2) = (200, -200, 800)/98$.]

Q11.2 A. [$\hat{\beta} = \frac{-200}{200} = -1$; $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 6 - (-1) \times 4 = 10 \Rightarrow y = 10 - x$.]

Q11.3 C. [$s^2 = \frac{1}{98}(800 - \frac{200^2}{200}) = \frac{600}{98} \approx 6$.]

Q11.4 B. [$r = \frac{-200}{\sqrt{200 \times 800}} = \frac{-200}{400} = -0.5$.]

Q11.5 D. [Definition]

Homework 11 answers:

1. A. [$n_1 = \binom{4}{1} = 4$, $n_2 = \binom{4}{2} = 6$, $n_3 = \binom{4}{3} = 4$, $n_4 = \binom{4}{4} = 1$.
(P, Q, R, S); (PQ, PR, PS, QR, QS, PS); (PQR, PQS, PRS, QRS); (PQRS).]
2. D. [There are $4 \times 8 = 32$ observations in all. $\hat{U} = \bar{Y}_{U1} - \bar{Y}_{U0}$, so $\text{var}(\hat{U}) = \frac{\sigma^2}{16} + \frac{\sigma^2}{16} = \frac{\sigma^2}{8}$, since there will be 16 observations at each level of U .]
3. C. [$\text{df}(U, V, UV, W) = 4$; $\text{df}(\text{blocks}) = 3$ and $\text{df}(\text{total}) = 31$, so $\text{df}(\text{error}) = 31 - 4 - 3 = 24$.]
4. B. [If the F-value for J is greater than the critical value, then this indicates that the effect of J is non-zero: it could be negative or positive.]
5. E. [$\hat{\beta} = \frac{50}{100} = 0.5$; $s^2 = \frac{1}{25}(125 - \frac{50^2}{100}) = 4$; $\text{se}(\hat{\beta}) = \sqrt{\frac{s^2}{K}} = \sqrt{0.04} = 0.2$.]
6. (a) Randomly order 1234 (representing treatments 1,2,3,4) within each block, using MATLAB or EXCEL (see text). For example: block 1: 3421; block 2: 2143; block 3: 1432.
- (b) i. The analysis of variance in this case is given by:

Source	DF	SS	MS	F	P
b	2	140.180	70.090	23.58	0.001
t	3	287.009	95.670	32.19	0.000
Error	6	17.833	2.972		
Total	11	445.023			

so that the treatment effects are significant: $F_T = 32.19$, $p = 0.000$.

- ii. Splitting up the treatments sum of squares gives the following analysis of variance:

Source	DF	SS	MS	F	P
b	2	140.180	70.090	23.58	0.001
p	1	222.741	222.741	74.94	0.000
q	1	60.301	60.301	20.29	0.004
p*q	1	3.967	3.967	1.33	0.292
Error	6	17.833	2.972		
Total	11	445.023			

- iii.
- $F_P = 74.94$
- ,
- $p = 0.000$
- indicating that
- P
- is highly significant.

iv. $\hat{P} = \text{av}(T_2 + T_4) - \text{av}(T_1 + T_3) = 60.8333 - 52.2667 = 8.617$. $\text{var}(\hat{P}) = \frac{\sigma^2}{6} + \frac{\sigma^2}{6}$,
so $\text{se}(\hat{P}) = \sqrt{\frac{s^2}{3}} = \sqrt{\frac{2.972}{3}} = 0.995$.