620.370 Statistics for Mechanical Engineers — Semester 2, 2009

Not Homework set 12

Tutorial problems: Quiz 12; 212, 214, 219.

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1. (a) The table below gives values of a variable x and a dependent variable y.

x	0	1	2	3	4	5
y	54.61	70.58	77.69	80.23	87.70	88.32
	65.44	68.12	72.16	75.75	86.67	
	54.83					

   i. Assuming that $\mathrm{E}(Y \mid x) = \alpha + \beta e^{-0.4x}$ and $\mathrm{var}(Y \mid x) = \sigma^2$, obtain estimates of α, β and σ^2 using the method of least squares.

   ii. Plot the observations and your fitted curve.

   iii. Find a prediction interval for an observation at x=5.

   (b) Consider the following bivariate data set:

x	31	46	41	52	36	47	53	36	27	58	39	50
y	82	60	87	63	73	50	55	76	80	42	70	49

   Assume that these data were obtained from a bivariate normal population with correlation ρ.

   i. Find the sample correlation coefficient, r.

   ii. Find a 95% confidence interval for ρ.

   iii. Test the hypothesis that X and Y are independent.

2. An investigation of a die-casting process resulted in the data below for x_1 = furnace temperature, x_2 = die close time, and y = temperature difference on the die surface ("A multiple-objective decision-making approach for assessing simultaneous improvement in die life and casting quality in a die casting process", *Quality Engineering*, 1994: 371–383).

x_1	1250	1300	1350	1250	1300	1250	1300	1350	1350
x_2	6	7	6	7	6	8	8	7	8
y	80	95	101	85	92	87	96	106	108

   Output from fitting the model $\eta = \mathrm{E}(y \mid x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ is given below:

```
Predictor      Coef   SE Coef       T      P
Constant    -199.56     11.64  -17.14  0.000
x1           0.2100   0.00864   24.30  0.000
x2           3.0000    0.4321    6.94  0.000
S = 1.05848   R-Sq = 99.1%   R-Sq(adj) = 98.8%

Analysis of Variance
Source          DF      SS      MS       F      P
Regression       2  715.50  357.75  319.31  0.000
Residual Error   6    6.72    1.12
Total            8  722.22
```

   (a) The model utility test is a test of β_1=β_2=0, i.e. whether the regression is significantly different from zero. Use the regression MS to test this hypothesis.

   (b) Calculate and interpret the 95% confidence interval for β_2.

   (c) When x_1=1300 and x_2=7, the standard error of $\hat{\eta}$ is 0.353. Calculate a 95% confidence interval for η when furnace temperature is 1300 and die close time is 7.

   (d) Calculate a 95% prediction interval for the temperature difference resulting from an experimental run with a furnace temperature of 1300 and a die close time of 7.

Quiz 12

Q12.1 If the correlation coefficient r_{XY} is not significantly different from zero, then this indicates that

A. X and Y are independent

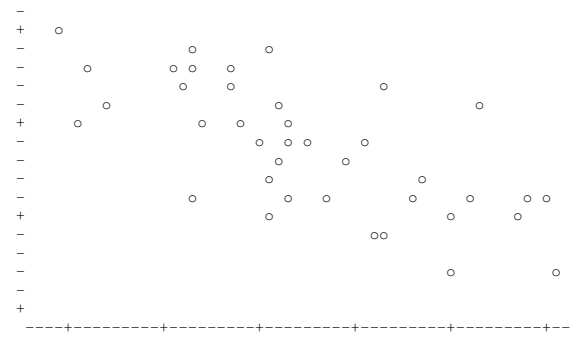
B. X and Y are not functionally related

C. X and Y are identically distributed

D. X and Y are uncorrelated

E. X and Y have equal means

Q12.2 For the data represented in the scatter diagram below, which <u>one</u> of the following statements about the correlation coefficient is true?



A. $0.6 < r < 1.0$

B. $0.2 < r < 0.6$

C. $-0.2 < r < 0.2$

D. $-0.6 < r < -0.2$

E. $-1.0 < r < -0.6$

Q12.3–12.4 refer to the information below:

The following Minitab output was obtained when a multiple regression model of the form $y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + e_i$, was fitted to a sample of 20 observations.

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The regression equation is
y = 16.6 + 0.76 x1 - 0.694 x2 + 5.64 x3

Predictor     Coef   SE Coef       T       P
Constant    16.578     4.292    3.86   0.001
x1           0.764     1.480    0.52   0.613
x2         -0.6941    0.4697   -1.48   0.159
x3           5.640     2.540    2.22   0.041

Analysis of Variance
SOURCE       DF        SS       MS      F      P
Regression    3   24393.2   8131.1  22.14  0.000
Error        16    5874.9    367.2
Total        19   30268.0
```

Q12.3 The value of R^2 is:

A. 0.649

B. 0.806

C. 0.194

D. 0.898

E. -0.898

Q12.4 A 95% confidence interval for β_2 is:

A. $(-1.690, 0.302)$

B. $(-1.695, 0.307)$

C. $(-1.514, 0.126)$

D. $(-1.164, -0.224)$

E. $(-0.799, -0.589)$