

Revision Exercises 1

- R1.1** (a) $\Pr(X \geq 3)$, where $X \stackrel{d}{=} \text{Bi}(4, 0.9)$
 $= 0.6561 + 0.2916 = 0.948$ (using tables).
- (b) $\Pr(Y = 0)$, where $Y \stackrel{d}{=} \text{Pn}(0.4)$
 $= 0.670$.
- (c) $F(t) = \int_0^t \frac{1}{(1+t)^2} dt = 1 - \frac{1}{1+t}$; $h(t) = \frac{1/(1+t)^2}{1/(1+t)} = \frac{1}{1+t}$.
- (d) $\text{var}(\ln X) \approx (\frac{1}{\mu})^2 \text{var}(X)$; $\text{sd}(\ln X) \approx \frac{1}{20} \times 2 = 0.1$.
- (e) $\frac{10S^2}{100} \stackrel{d}{=} \chi_{10}^2$; tables $\Rightarrow 32.47 < S^2 < 204.8 \Rightarrow 5.7 < S < 14.2$.
- (f) $n = 60, x = 48 \Rightarrow \hat{p} = 0.8$; Table 2 $\Rightarrow (0.68 < p < 0.89)$.
- (g) $\text{Bi}(100, 0.95)$
- (h) $\Pr(|Z| < c_q^*) = \Pr(-c_q^* < Z < c_q^*) = q$.
 Therefore $c_q^* = z_Q$, where $Q = q + \frac{1-q}{2} = \frac{1+q}{2}$, i.e. $c_q^* = \Phi^{-1}(\frac{1+q}{2})$.
 $c_{0.5}^* = z_{0.75} = 0.6745$.
- (i) $\frac{\partial \ln L}{\partial \theta} = -100(\theta - 1) = 0 \Rightarrow \hat{\theta} = 1$;
 $\frac{\partial^2 \ln L}{\partial \theta^2} = -100 \Rightarrow \text{se}(\hat{\theta}) = 1/\sqrt{100} = 0.1$.
- (j) $\hat{\sigma} = \frac{7.06}{3.532} = 2$; $\text{CL} = 14.5 \pm 3 \times \frac{2}{\sqrt{16}} = (13.0, 16.0)$.

R1.2 (a)

	A	A'	
B	0.08	0.12	0.2
B'	0.08	0.72	0.8
	0.16	0.84	1

- i. $\Pr(A) = 0.16$;
 ii. $\Pr(B|A) = 0.5$;
 iii. yes, $\Pr(A|B) > \Pr(A)$.
- (b) i. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.4 & 0.5 & 0 \\ 0 & 0.2 & 0.3 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- ii. $0.1166 + 0.1575 = 0.2741$;
 iii. 0.2188 ;
 iv. 0.9375 .

- R1.3** (a) $\mathbb{E}(X) = 0 \times 0.4 + 1 \times 0.3 + 2 \times 0.2 + 3 \times 0.1 = 0 + 0.3 + 0.4 + 0.3 = 1$;
 $\mathbb{E}(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.1 = 0 + 0.3 + 0.8 + 0.9 = 2$,
 so $\text{var}(X) = 2 - 1^2 = 1$.
- (b) $\Pr(X_1 = X_2) = 0.4^2 + 0.3^2 + 0.2^2 + 0.1^2 = 0.3$.
- (c) i. $D = S - T$: $\mathbb{E}(D) = \mathbb{E}(S) - \mathbb{E}(T) = 0$, $\text{var}(D) = \text{var}(S) + \text{var}(T) = 50 + 50 = 100$.
 ii. $S - T \stackrel{d}{\approx} N(0, 100)$; $\Pr(S = T) = \Pr(D = 0) \approx \Pr(-0.5 < D^* < 0.5)$;
 $\Pr(-0.5 < D^* < 0.5) = \Pr(-0.05 < D_s^* < 0.05) = 2 \times 0.0199 = 0.040$.

- R1.4** (a) i. $\bar{x} \approx 60$;
 ii. $s \approx 10$;
 iii. $\text{freq}(X < 50) \approx 100 \times 0.1587 \sim 16$;
 iv. $Q_3 \approx 60 + 0.6745 \times 10 = 66.7$;
 v. $\max = x_{(100)} \approx c_{0.99} = 60 + 2.33 \times 10 = 83.3$.
- (b) i. five number summary $\approx (37, 53, 60, 67, 83)$
 ii. QQ-plot: straight line with intercept ≈ 60 and slope ≈ 10 ; $(-2, 40) \rightarrow (2, 80)$.
 iii. same points, except that points corresponding to $x < 50$ are omitted.

- R1.5** (a) i. $\mathbb{E}(\bar{T}) = 100$, $\text{sd}(\bar{T}) = \sqrt{\frac{100^2}{400}} = 5$;
 ii. $100 \pm 2 \times 5 = (90, 110)$.
- (b) i. $\bar{x} = \frac{30.0}{5} = 6.0$;
 ii. $s^2 = \frac{1}{4}(200 - \frac{30^2}{5}) = 5$ or $s^2 = \frac{1}{4}(9 + 1 + 0 + 1 + 9) = 5$;
 iii. 95% CI for μ : $6 \pm 2.776 \times 1 = (3.2, 8.8)$;
 iv. 95% CI for σ : $(\sqrt{\frac{20}{11.14}}, \sqrt{\frac{20}{0.48}}) = (1.3, 6.5)$;
 v. 95% PI for X : $6 \pm 2.776 \times 6 = (-0.8, 12.8)$.
- R1.6** (a) i. $t = \frac{44.8 - 50}{8/\sqrt{16}} = \frac{-5.2}{2} = -2.6$, cf. $c_{0.975}(t_{15}) = 2.131$; hence H_0 is rejected;
 ii. $P = 2 \Pr(t_{15} > 2.6) \approx 0.02$.
- (b) Reject the null hypothesis $\mu=50$. There is significant evidence indicating that the mean strength of adhesion is less than 50.
- R1.7** (a) i. $s^2 = \frac{14 \times 60 + 9 \times 44.7}{23} = 54.0$
 ii. 95% CI for $\mu_1 - \mu_2$: $-6.5 \pm 2.069 \sqrt{54(\frac{1}{15} + \frac{1}{10})} = (-6.5 \pm 6.2) = (-12.7, -0.5)$.
- (b) i. $s^2 = \frac{1}{3}(7.0 + 10.0 + 7.0) = 8$;

B	2	78	39	4.88*
W	9	72	8	
T	11	150		

 ii. $F = 4.88$, cf. $c_{0.95}(F_{2,9}) = 4.26$; reject H_0 , $P \approx 0.04$.
 iii. 95% CI for $\mu_R - \mu_Q$: $1.5 \pm 2.262 \sqrt{8(\frac{1}{4} + \frac{1}{4})} = (1.5 \pm 4.5) = (-3.0, 6.0)$.
 iv. $P < R \approx Q$.
- R1.8** (a) balance (twice in each block); randomisation (within blocks).
- (b) i.

B	3	400	133	
T	2	84	42	8.4
e	18	90	5	
y	23	574		

 ii. $F_T = 8.4$, cf. $c_{0.95}(F_{2,18}) = 3.56$; the treatment effects are significant.
 iii. $\hat{\mu}_A - \hat{\mu}_C = 20 - 10 = 10$; $\text{se}(\hat{\mu}_A - \hat{\mu}_C) = \sqrt{5(\frac{1}{8} + \frac{1}{8})} = 1.12$.
- R1.9** i. effect zero $\Rightarrow \sqrt{SS} \stackrel{d}{=} \sigma|N|$, so points not on the half-normal line indicate non-zero effects. In this case, D , A , C and CD are away from the line through the rest of the points.
- ii.

A	1	169	
C	1	81	
D	1	225	
CD	1	16	cf. $c_{0.95}(F_{1,11}) = 4.84$
e	11	13	
y	15	504	
- iii. $s^2 = \text{error MS} = \frac{13}{11}$;
 iv. $\hat{A} = 6.5$;
 95% CI for A : $6.5 \pm 2.2 \times 1.1 \times \sqrt{\frac{1}{8} + \frac{1}{8}} = (5.3, 7.7)$.
- v. D has a greater effect at high level of C (and C has a greater effect at high level of D).
- R1.10** i. $\hat{\beta} = \frac{180}{100} = 1.8$; $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 30 - 18 = 12$.
 ii. $s^2 = \frac{1}{8}(400 - \frac{180^2}{100}) = \frac{76}{8} = 9.5$; $\text{se}(\hat{\beta}) = \frac{\sqrt{9.5}}{10} = 0.31$;
 iii. (scatter-plot and fitted line: $y = 12 + 1.8x$)
 iv. $r = \frac{180}{10 \times 20} = 0.9$;
 95% CI for ρ : $(0.60, 0.97)$ (using Table 20).

Revision Exercises 2

R2.1 (a) i. $\Pr(G \cup H) = 0.6$;

ii. $\Pr(G \cup H) = 0.4 + 0.2 - 0.4 \times 0.2 = 0.52$;

iii. $\Rightarrow \Pr(G \cup H) = 0.44.$

	H	H'	
G	0.16	0.24	0.4
G'	0.04	0.56	0.6
	0.2	0.8	1

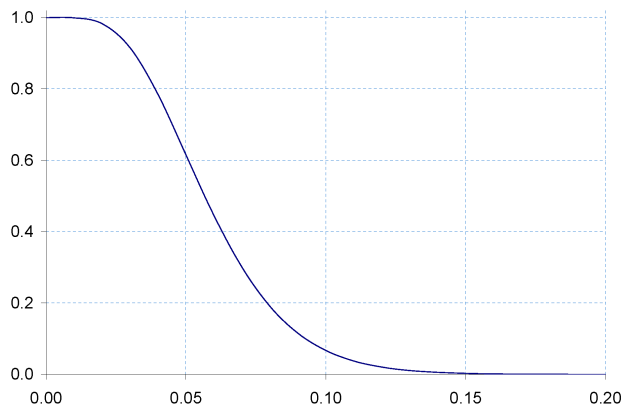
(b) i. $p_0 = 0.5 \times 0.4 \times 0.2 = 0.04$; $\Pr(\text{at least one}) = 0.96$;

ii. $p_1 = 0.5 \times 0.4 \times 0.2 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.4 \times 0.8 = 0.26$;

$\Pr(\text{at least two}) = 1 - 0.04 - 0.26 = 0.70.$

R2.2 number of defectives, $X \stackrel{d}{=} \text{Bi}(100, p) \approx \text{Pn}(100p).$

$P_a = \Pr(X \leq 5)$; $P_a(p=0) = 1$, $P_a(p=0.05) = 0.616$, $P_a(p=0.1) = 0.067.$



R2.3 (a)
$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.05 & 0.25 & 0.7 & 0 & 0 \\ 0.05 & 0 & 0.25 & 0.7 & 0 \\ 0.05 & 0 & 0 & 0.25 & 0.7 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) i. $0.0010 + 0.0137 + 0.0766 = 0.0913$;

ii. $0.18 = 18\%$ [using p_{10}];

iii. 0.87 [using $1 - p_{20}$].

R2.4 (a) $X \stackrel{d}{=} \text{Pn}(1.2) \Rightarrow \Pr(X = 0) = 0.301$;

(b) $X \stackrel{d}{=} \text{Bi}(18, 0.1) \Rightarrow \Pr(X \leq 2) = 0.734$;

(c) $X \stackrel{d}{=} \text{N}((-0.05, 0.1^2)) \Rightarrow \Pr(-0.2 < X < 0.2) = \Pr(-1.5 < X_s < 2.5) = 0.927$;
probability of rejection = 0.073.

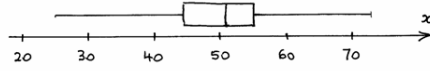
R2.5 i. $\bar{X} \stackrel{d}{=} \text{N}(0, \frac{1}{16}) \Rightarrow \Pr(\bar{X} > 0.5) = \Pr(\bar{X}_s > 2) = 0.023$;

ii. $15S^2 \stackrel{d}{=} \chi_{15}^2 \Rightarrow \Pr(S^2 > 2) = \Pr(\chi_{15}^2 > 30) \approx 0.01$;

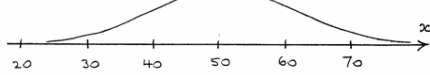
iii. $\mathbb{E}(X_{(16)}) = 1.766$;

iv. $\Pr(X_{(16)} \leq 1.2816) = \Pr(X_1, \dots, X_{16} \leq 1.2816)$
 $= \Pr(Z = 16)$, where $Z \stackrel{d}{=} \text{Bi}(16, 0.9)$
 $= 0.185.$

- R2.6** i. $\hat{m} = 51$;
 ii. five number summary: (25, 44, 51, 55, 73);



- iii. pdf graph approx $N(50, 10^2)$: bell-shaped between roughly 30 and 70;



- iv. $\sigma \approx 10$ [using $\frac{1}{4}(c_{0.975} - c_{0.025})$]
 v. a probability plot is a sample cdf but with a warped vertical scale, warped in such a way that a (normal) cdf is a straight line.

- R2.7** (a) $\hat{\lambda} = \bar{y} = 7$; $se(\hat{\lambda}) = \sqrt{\frac{\hat{\lambda}}{n}} = \sqrt{\frac{7}{10}} = 0.84$.
 (b) Table 2 $\Rightarrow (0.19 < p < 0.43)$.
 (c) i. 90% CI for μ : $(50 \pm 1.761 \times \sqrt{60 \times \frac{1}{15}}) = (50 \pm 3.5) = (46.5, 53.5)$;
 ii. 90% PI for X : $(50 \pm 1.761 \times \sqrt{60 \times \frac{16}{15}}) = (50 \pm 14.0) = (36.0, 54.0)$.
 (d) $\frac{UCL}{LCL} = (8.2 \pm 3\sqrt{\frac{1.6}{10}}) = (8.2 \pm 1.2) = (7.0, 9.4)$.

- R2.8** (a) $\ln L = n \ln \theta - (\theta + 1) \sum \ln(1 + x_i)$;
 $\frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta} - \sum \ln(1 + x_i)$; $\frac{\partial \ln L}{\partial \theta} = 0 \Rightarrow \hat{\theta} = \frac{n}{\sum \ln(1 + x_i)}$;
 $\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{n}{\theta^2} \Rightarrow se(\hat{\theta}) = \frac{\hat{\theta}}{\sqrt{n}}$.
 (b) i. $F(c_q) = q \Rightarrow 1 - q = \frac{1}{(1 + c_q)^\theta} \Rightarrow -\ln(1 - q) = \theta \ln(1 + c_q)$;
 ii. $c_q \sim x_{(k)}$, where $q = \frac{k}{n+1}$, so $-\ln(1 - \frac{k}{n+1}) \sim \theta \ln(1 + x_{(k)})$.

If the model is correct then the plot of $\ln(1 + x_{(k)})$ vs z_k should be close to a straight line through the origin; its slope will give an estimate of $1/\theta$.

- R2.9** Let $V = S^2$. Then $\mathbb{E}(V) = \sigma^2$ and $\text{var}(V) = \frac{2\sigma^4}{n-1}$.
 $\text{var}(S) \approx (\frac{1}{2\sqrt{\mu_V}})^2 \text{var}(V) = \frac{1}{4\sigma^2} \frac{2\sigma^4}{n-1} \Rightarrow se(s) = \frac{s}{\sqrt{2n-2}}$.
 $n = 33, s = 16 \Rightarrow se(s) = \frac{16}{\sqrt{64}} = 2$.
 approx 95% CI for σ : $(16 \pm 4) = (12, 20)$.
 Using $\frac{32S^2}{\sigma^2} \stackrel{d}{=} \chi_{32}^2$, we have $\Pr(18.29 < \frac{32S^2}{\sigma^2} < 49.48) = 0.95$, and hence:
 exact 95% CI for σ : $(16\sqrt{\frac{32}{49.48}}, 16\sqrt{\frac{32}{18.29}})$

- R2.10** i. $\hat{\theta} = 2.5$ [where $\ln L$ is a maximum];
 ii. $\{\ln L \geq -2\} = (1.6 < \theta < 3.7)$;
 iii. $\frac{\partial^2 \ln L}{\partial \theta^2} \Big|_{\hat{\theta}} = -4 \Rightarrow se(\hat{\theta}) = 1/\sqrt{4} = 0.5$.
Note: another approximate 95% CI is $(2.5 \pm 2 \times 0.5) = (1.5, 3.5)$.

- R2.11** (a) size = $\Pr(\text{reject } H_0 | H_0)$, power = $\Pr(\text{reject } H_0 | H_1)$.
 (b) reject H_0 if $X_{300} \geq 22$;
 this test has size 0.0486 ($= 1 - 0.9514$), and power 0.9542 ($= 1 - 0.0458$).
 (c) i. if we accept H_1 , then
 $U_k = \ln \Pr(\text{accept } H_1 | H_1) - \ln \Pr(\text{accept } H_1 | H_0) \approx \ln 0.95 - \ln 0.05 \approx 3$;
 and if we accept H_0 , then
 $U_k = \ln \Pr(\text{accept } H_0 | H_1) - \ln \Pr(\text{accept } H_0 | H_0) \approx \ln 0.05 - \ln 0.95 \approx -3$.
 ii. $L = c\theta^{x_k}(1-\theta)^{k-x_k} \Rightarrow \ln L = c + x_k \ln \frac{\theta}{1-\theta} + k \ln(1-\theta)$;
 $\therefore U_k = x_k \ln\left(\frac{0.1}{0.9} \frac{0.95}{0.05}\right) + k \ln \frac{0.9}{0.95} \approx 0.75x_k - 0.05k$;
 iii. $k = 200$, $x_k = 20 \Rightarrow U_k \approx 5$;
 iv. advantages: likely to give a quicker/cheaper result;
 disadvantage: more administration.

- R2.12** paired/matched samples \Rightarrow consider the sample of differences
 t-test: $t = \frac{\bar{d}}{s_d/\sqrt{n}}$.
 all $d > 0$; dotplot suggests that $\mu_d > 0$, i.e. $\mu_N > \mu_D$.

- R2.13** i. $\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \stackrel{d}{=} F_{7,7} \Rightarrow \Pr\left(\frac{1}{5} < \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} < 5\right) = 0.95$
 95% CI: $\frac{1}{5} \times \frac{40}{32} < \frac{\sigma_1^2}{\sigma_2^2} < 5 \times \frac{40}{32} \Rightarrow \frac{1}{2} < \frac{\sigma_1}{\sigma_2} < \frac{5}{2}$;
 since $1 \in \text{CI}$, we accept ($\sigma_1 = \sigma_2$).
 ii. $s^2 = \frac{1}{2}(s_1^2 + s_2^2) = 36$.
 iii. 95% CI for $\mu_1 - \mu_2$: $(30 \pm 2.145\sqrt{36(\frac{1}{8} + \frac{1}{8})}) = (30 \pm 6.4) = (23.6, 36.4)$.

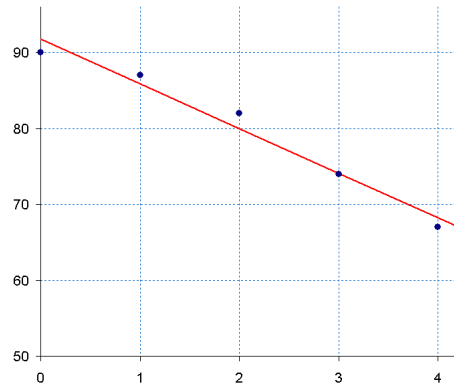
- R2.14** (a) apply the four treatments once within each block; randomly allocate the treatments to the units within each block.
 (b) i.

blocks	4	400	100	
treatments	3	600	200	13.3
error	12	180	15	
total	19	1180		

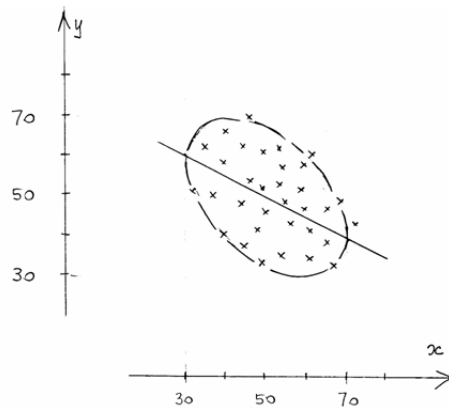
 ii. $F = 13.3$, cf. $c_{0.95}(F_{3,12}) = 3.49$,
 so there is a significant difference between the treatment effects.
 iii. $F_P = \frac{P}{e} \frac{MS}{MS} = \frac{500}{15} = 33.3$, cf. $c_{0.999}(F_{1,12}) = 18.64$.
 iv. $\hat{P} = \frac{250}{10} - \frac{150}{10} = 10$; $se(\hat{P}) = \sqrt{15(\frac{1}{10} + \frac{1}{10})} = \sqrt{3} = 1.73$

- R2.15** i. on a half-normal plot for \sqrt{SS} , the values for the “zero” effects will follow a straight line (indicating that they are just noise), the non-zero effects (the signal) stand out away from the line;
 the “zero” effects are seen as noise and they are combined with the error term: in this case all the interaction terms are seen as noise, leaving only the main effects (P, Q, R, S) in the model fitted in the second analysis of variance.
 P, Q and S are significant; $s^2 = 3.60$.
 ii. $\hat{P} = 3.23$;
 95% CI for P : $(3.23 \pm 2.20 \times \sqrt{\frac{3.6}{4}}) = (3.23 \pm 2.09) = (1.14, 5.32)$.

- R2.16** (a) i. $\hat{\beta} = -\frac{59}{10} = -5.9$, $\hat{\alpha} = 80 + 2 \times 5.9 = 91.8$;
 ii. $s^2 = \frac{1}{5-2} (358 - \frac{59^2}{10}) = 3.3$; $se(\hat{\beta}) = \sqrt{0.33} = 0.57$;
 iii.



- (b) i.
 ii.



iii. Table 20 $\Rightarrow (-0.68 < \rho < -0.27)$

Revision Exercises 3

R3.1. (a)

	B	\bar{B}	
A	0.3	0.2	0.5
\bar{A}	0.1	0.4	0.5
	0.4	0.6	1

$\Pr(B | \bar{A}) = \frac{0.1}{0.5} = 0.2$.

(b)

	F	\bar{F}	
O	0.009	0.9	
m	0.048	0.08	
M	0.019	0.02	
	0.076		

$\Pr(F) = 0.076 = 7.6\%$; $\Pr(m | F) = \frac{0.019}{0.076} = 0.25$.

R3.2. (a) $X \stackrel{d}{=} \text{Bi}(400, 0.1)$; $\mu = 40$, $\sigma^2 = 36$; approx 95% interval: $40 \pm 2 \times 6 = (28 \leq X \leq 52)$.

(b) $X \stackrel{d}{\approx} \text{Pn}(100p)$:

p	0	0.01	0.02	0.03	0.04	0.05
$\Pr(X \leq 2)$	1	0.92	0.68	0.42	0.24	0.13

R3.3. (a) i. $P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.7 & 0 & 0 \\ 0.1 & 0 & 0.2 & 0.7 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$;

ii. $\mathbb{E}(N) = \frac{1}{0.8} + \frac{1}{0.8} + \frac{1}{0.4} = 5$; **iii.a.** 0.233, **b.** 0.533, **c.** 0.234.

R3.4. (a)i. $0.65 - 1 = 0.55$, $1 - 0.65 = 0.35$; **ii.** 0.42; 0.30, 0.56.

(b) $\mathbb{E}(T) = \int_0^1 (12t^3 - 12t^4) dt = \frac{12}{4} - \frac{12}{5} = 0.6$; $\mathbb{E}(T^2) = \int_0^1 (12t^4 - 12t^5) dt = \frac{12}{5} - \frac{12}{6} = 0.4$;
 so $\text{var}(T) = 0.4 - 0.6^2 = 0.04$, and hence $\text{sd}(T) = 0.2$.

- R3.5.** (a)i. $\Pr(X > 12) = \Pr(Z > 0.5) = 0.3085$;
 ii. $X - Y \stackrel{d}{=} N(-2, 5^2)$; $\Pr(X - Y > 0) = \Pr(Z > 0.4) = 0.3446$.
 (b) $\Pr(Z \geq 33) \approx \Pr(Z^* > 32.5) = \Pr(Z > 1.5) = 0.0668$;
 (c) $\mathbb{E}(\ln U) \approx 2.303 - \frac{5^2}{2 \times 10^2} = 2.18$; $\text{var}(\ln U) \approx \frac{5^2}{10^2} \Rightarrow \text{sd}(\ln U) \approx 0.5$.
- R3.6.** (a)i. 3; ii. 6; iii. $s^2 = \frac{1}{9}(432 - 360)$, $s = \sqrt{8} = 2.8$; iv. 6;
 v. $\frac{11}{4} = 3.75 \rightarrow Q1 = 4$; $\frac{33}{4} = 8.25 \rightarrow Q1 = 8.25$.
 (b) $\hat{\lambda} = \bar{y} = 6$; $\text{se}(\bar{y}) = \sqrt{\frac{6}{10}} = 0.77$.
- R3.7.** (a) $0.15 < p < 0.49$ (from diagram in Tables); [est $\pm 2 \times \text{se}$ gives $0.13 < p < 0.47$.]
 (b)i. $160 \pm 1.245 \sqrt{\frac{60}{15}} = 160 \pm 4.3 = (155.7, 164.3)$; $160 \pm 1.245 \sqrt{60 \times \frac{16}{15}} = 160 \pm 17.2 = (143, 177)$.
 (c) $\frac{\partial \ln L}{\partial \theta} = \frac{200}{\theta} - 50\theta = 0 \Rightarrow \hat{\theta} = 2$; $\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{200}{\theta^2} - 50 = 0 \Rightarrow \text{se}(\hat{\theta}) = \frac{1}{\sqrt{100}} = 0.1$.
- R3.8.** (a) Any of the following: $z = \frac{46.4 - 50}{10/5} = -1.8 < -1.645$; $46.4 + 1.645 \times \frac{10}{5} = 49.7 < 50$;
 or $P = 0.036 < 0.05$. So we reject H_0 , and say that “the mean is significantly less than 50”.
 (b)i. tables $\Rightarrow \mathbb{E}(X_{(25)}) = 1.965\sigma$, so $\mathbb{E}(R) = 3.93\sigma$, and hence $\bar{R} \approx 3.93\sigma$.
 ii. $\text{LCL} = 22.0 - 3 \times \frac{7.86}{3.93} \times \frac{1}{\sqrt{25}} = 22.0 - 1.2 = 20.8$; $\text{UCL} = 22.0 + 1.2 = 23.2$.
- R3.9.** i. $F_{7,7}$: $0.2 < \frac{S_2^2}{S_1^2} \frac{\sigma_1^2}{\sigma_2^2} < 5$, so a 95% CI is given by $\frac{1}{2} < \frac{\sigma_1}{\sigma_2} < \frac{5}{2}$; and $\frac{\sigma_1}{\sigma_2} = 1 \in \text{CI}$, so we accept H_0 .
 ii. $s^2 = \frac{1}{2}(s_1^2 + s_2^2) = 36$; $\frac{14s^2}{\sigma^2} \stackrel{d}{=} \chi_{14}^2$.
 iii. $30 \pm 2.145 \times 6 \times \sqrt{\frac{1}{8} + \frac{1}{8}} = 30 \pm 6.435 = (23.56, 36.44)$.
- R3.10.** i.

B	3	300	100	4
W	13	325	25	
T	16	625		

 ii. $F = 4 > c_{0.95}(F_{3,13}) \approx 3.46$; and so we reject H_0 .
 iii. $s^2 = 25$, $\frac{13s^2}{\sigma^2} \stackrel{d}{=} \chi_{13}^2 \Rightarrow 5 < \frac{13s^2}{\sigma^2} < 25$, which gives $13 < \sigma^2 < 65$.
 iv. $31 \pm 2.160 \times 5 \times \frac{1}{\sqrt{4}} = 31 \pm 5.4 = (25.6, 36.4)$.
 v. $\text{se} = \sqrt{25(\frac{1}{5} + \frac{1}{5})} = \sqrt{10} = 3.16$.
- R3.11.** (a) allocate each treatment twice at random within each block;
- | | | | | |
|---|----|-----|----|---|
| B | 2 | 48 | 24 | 8 |
| T | 2 | 24 | 12 | 4 |
| E | 13 | 39 | 3 | |
| Σ | 17 | 111 | | |
- Since $F = 4 > 3.81$, the treatment effects are significantly different from zero; $\text{se} = \sqrt{3(\frac{1}{6} + \frac{1}{6})} = 1$.
- (b)

temp	2	ns
catalyst	2	*
interaction	4	*
error	27	
- R3.12.** i. $r = 0.5$; $0.02 < \rho < 0.78$ (using the diagram in the Tables);
 ii. $\hat{\beta} = \frac{200}{400} = 0.5$; $\hat{\alpha} = 50 - 0.5 \times 30 = 35$;
 iii. (scatter plot)
 iv.

reg	1	100	100
res	15	300	20
total	16	400	

 $F = 5 > c_{0.95}(F_{1,15}) = 4.5$; so reject $\beta = 0$.