

620-371: Linear Models

Assignment 1

Due: Monday, 30th March, 2009

This assignment is worth 5% of your total mark. Fill in a plagiarism declaration form and hand it in together with this assignment.

1. Let A_1, A_2, \dots, A_m be a collection of symmetric $k \times k$ matrices. Suppose that all A_i , $i = 1, 2, \dots, m$ are idempotent, and that $A_i A_j = 0$ for all $i \neq j$. Show that $\sum_{i=1}^m A_i$ is an idempotent matrix.

2. Let

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, A = \frac{1}{6} \begin{bmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix}, B = \begin{bmatrix} -2 & 5 & 3 \\ 5 & 1 & -4 \\ 3 & -4 & 0 \end{bmatrix}.$$

Suppose that

$$E[\mathbf{y}] = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}, \text{var } \mathbf{y} = V = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

- (a) Find $E[\mathbf{y}^T A \mathbf{y}]$.
 - (b) Describe the distribution of $\frac{1}{4} \mathbf{y}^T A \mathbf{y}$.
 - (c) Are $\mathbf{y}^T A \mathbf{y}$ and $\mathbf{y}^T B \mathbf{y}$ independent?
3. A study is conducted to determine if (and how) the fuel mileage of a car is dependent on its weight, and the speed at which it is driven. A linear model is assumed, and the following data is obtained:

Mileage (km/litre)	Weight (tons)	Speed (km/hr)
8.5	1.35	34
8	1.33	36
7.5	2	38
10	1.4	34
11	1.4	31
15	1.2	31
13.5	1.3	33
14.5	1.28	41

- (a) Write down the linear model in matrix form.
- (b) Write down the normal equations for this model.

(c) Solve the normal equations to estimate the parameters.

You may use R for this question, but for the matrix calculations only. If you do, include your R commands and output.