620-371: Linear Models

Assignment 2

Due: Monday, 4th May, 2009

This assignment is worth 5% of your total mark.

You may use R (and only R) for this assignment, but for matrix calculations and statistical distributions only. You may not use the `lm` function. If you do use R, include a computer printout of your commands and R output.

An experiment is conducted to estimate the demand for cars, based on their cost, the current unemployment rate, and the current interest rate. A survey is conducted and the following measurements obtained:

Cost ($\$k$)	Unemployment rate (%)	Interest rate (%)	Cars sold ($\times 10^3$)
9.0	10.0	4.0	6.5
5.5	9.0	7.0	5.9
9.0	12.0	5.0	8.0
9.8	11.0	6.2	9.0
14.5	12.0	5.8	10.0
8.0	13.7	3.9	10.8

1. (a) Calculate the least squares estimator, $\mathbf{b}$.
   (b) In this year, the unemployment rate is 7% and the interest rate is 8%. What price should a dealer put on a car to expect to sell 7500 cars?
   (c) Find a 95% confidence interval for the average number of $\$10,000$ cars sold in a year which has unemployment rate 4.5% and interest rate 7%.

2. (a) Test for model adequacy.
   (b) We want to find the best model for this data. Starting with an empty model, perform tests at the 5% level to determine if each parameter should be added to the model, in the order intercept, cost, unemployment, interest rate. If a parameter should be added, add it for the remaining tests; otherwise, leave it out. Using this approach, what is the best model?
   (c) Simultaneously test the hypotheses that the unemployment rate is not relevant to the cars sold and that the parameters corresponding to cost and interest rate are equal.

3. We can derive a joint confidence region for the parameters of a full rank linear model,
$$(\mathbf{b} - \boldsymbol{\beta})^T X^T X (\mathbf{b} - \boldsymbol{\beta}) \leq p s^2 f_\alpha.$$

Use this joint confidence region to derive a test for the hypothesis $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}^*$. Show that this test is equivalent to the existing test for $H_0 : \boldsymbol{\beta} = \boldsymbol{\beta}^*$.