620-371: Linear Models

Assignment 3

Due: Monday, 25th May, 2009

This assignment is worth 5% of your total mark.

You may use R (and only R) for this assignment, but only for questions 1 and 2e. You may not use the `lm` function for question 1, but you may use it for question 2e. If you do use R, include a computer printout of your commands and R output.

1. We are interested in examining the yield of tomato plants that have been grown with certain types of fertiliser. A study is conducted and the following data obtained:

   Fertiliser

1	2	3
43	33	54
45	37	54
47	38	57
46	35	
48		

   We fit the model
   $$y_{ij} = \mu + \tau_i + \varepsilon_{ij},$$
   where μ is the overall mean and τ_i is the effect of using the ith fertiliser.

   (a) Find a conditional inverse for $X^T X$, using the algorithm given in the lecture slides.

   (b) Find two solutions to the normal equations, using only the conditional inverse you found in question 1a.

   (c) Is $\mu + \tau_1 - \tau_2 + \tau_3$ estimable?

   (d) Find a 95% confidence interval for the estimable quantity $\tau_2 - \tau_3$.

   (e) Test the hypothesis that fertiliser has no effect on yield.

2. We study the amount of rotting of a potato exposed to a variety of levels of oxygen, and a variety of temperatures. A small experiment is conducted and the following data obtained:

	Oxygen level		
Temperature	1	2	3
10	13	10	15
	11	4	2
	3	7	7
16	26	15	20
	19	22	24
	24	18	8

We fit the model

$$y_{ijk} = \mu + \tau_i + \beta_j + \varepsilon_{ijk}, \tag{1}$$

where μ is the overall mean, τ_i is the effect of the ith oxygen level, and β_j is the effect of the jth temperature level. A model was fitted in R using the `lm` function, from which the following output was derived:

```
> options(contrasts = c("contr.treatment", "contr.poly"))
> model <- lm(rot ~ oxygen.f + temp.f, data = potato)
> summary(model)

Call:
lm(formula = rot ~ oxygen.f + temp.f, data = potato)

Residuals:
     Min       1Q   Median       3Q      Max
-10.4444  -2.8611   0.4444   3.0278   8.1111

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   10.222      2.410   4.242 0.000820 ***
oxygen.f2     -3.333      2.951  -1.130 0.277660
oxygen.f3     -3.333      2.951  -1.130 0.277660
temp.f16      11.556      2.410   4.796 0.000285 ***
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Residual standard error: 5.111 on 14 degrees of freedom
Multiple R-squared: 0.6382,       Adjusted R-squared: 0.5607
F-statistic: 8.233 on 3 and 14 DF,  p-value: 0.002105

> anova(model)

Analysis of Variance Table

Response: rot
          Df Sum Sq Mean Sq F value    Pr(>F)
oxygen.f   2  44.44   22.22  0.8505 0.4481124
temp.f     1 600.89  600.89 22.9988 0.0002849 ***
```

```
Residuals 14 365.78   26.13
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

> linear.hypothesis(model, c(0, 1, 0, -1), 0)

Linear hypothesis test

Hypothesis:
oxygen.f2 - temp.f16 = 0

Model 1: rot ~ oxygen.f + temp.f
Model 2: restricted model

  Res.Df     RSS Df Sum of Sq      F   Pr(>F)
1     14  365.78
2     15  764.80 -1   -399.02 15.272 0.001577 **
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

(a) What information do you gather about the parameters of the model in equation 1, from the coefficients of the fitted R model?

(b) Should we accept or reject the hypothesis that oxygen level has no effect on rot?

(c) Should we accept or reject the hypothesis that temperature has no effect on rot?

(d) What hypothesis is the `linear.hypothesis` function testing, given in terms of the parameters of the model in equation 1?

(e) Using R, test for the presence of interaction between the factors.

3. Show that for any matrix A, the matrix $A(A^TA)^cA^T$ is invariant to the choice of a conditional inverse (i.e. unique). You may use the property of a general matrix M that if $M^TM = 0$, then $M = 0$. (*Hint: Let* $(A^TA)^c_1$ *and* $(A^TA)^c_2$ *be two different conditional inverses for* A^TA *and show that* $A(A^TA)^c_1A^T - A(A^TA)^c_2A^T = 0$.)