620-371: Linear Models

Practice Class 2

10th March, 2009

1. Show that $X^T X$ is a symmetric matrix.

2. (a) Let
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
be a nonsingular 2×2 matrix. Show by direct multiplication that
$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

   (b) Find the inverse of
$$\begin{bmatrix} 2 & 4 \\ 1 & -3 \end{bmatrix}.$$

3. Is
$$X = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
orthogonal? If not, what value of c makes the matrix cX orthogonal?

4. (a) Find the eigenvalues, and an associated eigenvector for each eigenvalue, of the matrix
$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}.$$

   (b) Find an orthogonal matrix P such that $P^T AP$ is diagonal.

   (c) Write down $P^T AP$ for the P given in part (b).

5. Let
$$A = \begin{bmatrix} 1 & 4 & 3 \\ -2 & 0 & 2 \\ 4 & 4 & 0 \end{bmatrix}.$$

   (a) Write down the trace of A.

   (b) Are the columns of A linearly independent? Justify your answer.

   (c) Find the rank of A.

6. Show that if X is of full rank, then
$$I - X(X^T X)^{-1} X^T$$
is an idempotent matrix.