620-371: Linear Models

Practice Class 4: Bring Your Calculators!

24th March, 2009

This practice class will use a single example. We model an individual's income at age 30 against the number of years of formal education (with a linear model). The following data is collected:

Years of formal education (x)	Income (\$k) (y)
8	8
12	15
14	16
16	20
16	25
20	40

1. Write down the linear model in matrix form.
2. Find the normal equations for this model.
3. Estimate the parameters using the least squares method.
4. This model is a simple linear regression model. Use the standard linear regression formulae,

$$b_1 = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2}, b_0 = \bar{y} - b_1\bar{x},$$

   to estimate the parameters again (where the bar indicates the mean). Check that you have the same answers as in (c).
5. Estimate the income of a person who has had 18 years of formal education.
6. We know that the least squares estimator $\mathbf{b}$ is an unbiased estimator for $\boldsymbol{\beta}$. Show that $\mathbf{t}^T\mathbf{b}$ is an unbiased estimator for $\mathbf{t}^T\boldsymbol{\beta}$, where $\mathbf{t}$ is a vector of constants.