620-371: Linear Models

Practice Class 6

7th April, 2009

1. Derive the formula for the confidence interval of the individual parameter β_0 from the formula for the confidence interval of a linear combination of parameters:
$$\mathbf{t}^T\mathbf{b} \pm t_{\alpha/2} s \sqrt{\mathbf{t}^T (X^T X)^{-1} \mathbf{t}}.$$

2. We model the energy consumption of a household in terms of the household income. The data we collect is:

Income (\$ k)	Energy consumption ($\times$10 Btu/yr)
20	1.8
30	3.0
40	4.8
55	5.0
60	6.5

   - Find a 95% confidence interval for the average energy consumption of households with yearly income \$50,000. You may use $t_{0.025} = 3.182$ for 3 degrees of freedom.
   - Find a 95% prediction interval for the energy consumption of a randomly selected household with yearly income \$50,000.

3. Prove that
$$\frac{(\mathbf{b} - \boldsymbol{\beta})^T X^T X (\mathbf{b} - \boldsymbol{\beta})}{\sigma^2}$$
has a χ^2 distribution with p degrees of freedom. *(Hint: you will need a corollary on the distribution of a quadratic form).*

4. Using the data from question 2, find a joint 95% confidence region for the two parameters β_0 and β_1. You may keep your answer as an implicit inequality, and use $f_{0.05} = 9.55$ for 2 and 3 degrees of freedom.

5. We can write SS_{Reg} and SS_{Res} as
$$SS_{Reg} = \mathbf{y}^T X (X^T X)^{-1} X^T \mathbf{y}$$
and
$$SS_{Res} = \mathbf{y}^T [I - X(X^T X)^{-1} X^T] \mathbf{y}.$$
Show that these two quadratic forms are independent.

6. Using the data from question 2, test the null hypothesis $H_0 : \boldsymbol{\beta} = \mathbf{0}$.