620-371: Linear Models

Practice Class 9

5th April, 2009

1. Show that in a mutually orthogonal full rank model, X^TX is diagonal. Calculate $(X^TX)^{-1}$ (expressed in terms of the elements of the X matrix).

2. Consider a less than full rank model with two factors. Factor 1 has two levels, and factor 2 has 3 levels. We take 2 samples from each possible combination of factor levels. We denote the response variable from the kth sample from the combination of factors with the first factor at level i and the second factor at level j to be y_{ijk}. We also denote the overall mean by μ, and assume that each level of each factor adjusts this overall mean by a certain amount: τ_i for the ith level of factor 1, and β_j for the jth level of factor 2.

   (a) Express y_{ijk} according to μ, τ_i, β_j, and an error term.

   (b) Write down the linear model in matrix form.

3. Let
$$A = \begin{bmatrix} 1 & 2 & 5 & 2 \\ 3 & 7 & 12 & 4 \\ 0 & 1 & -3 & -2 \end{bmatrix}.$$

   (a) Show that $r(A) = 2$.

   (b) Find a conditional inverse for A.

4. Show that $A = A(A^TA)^cA^TA$. You may use the result that if $A^TA = 0$, then $A = 0$. (*Hint: Consider the matrix* $A - A(A^TA)^cA^TA$).

5. It is known that toxic material was dumped into a river that flows into a large salt-water commercial fishing area. We are interested in the amount of toxic material (in parts per million) found in oysters harvested at three different locations in this area. A study is conducted and the following data obtained:

Site 1	Site 2	Site 3
15	19	22
26	15	26

   (a) Write down the linear model in matrix form.

   (b) Write down the normal equations.

   (c) Find a conditional inverse for X^TX.

   (d) Find a solution for the normal equations.