

# Linear Models: R Examples — Linear algebra

# Defining a matrix

```
> A <- c(1, 2, 0, 2, 3, -1, 0, -1, 8)
```

```
> dim(A) <- c(3, 3)
```

```
> A
```

```
      [,1] [,2] [,3]
[1,]    1    2    0
[2,]    2    3   -1
[3,]    0   -1    8
```

```
> A <- matrix(c(1, 2, 0, 2, 3, -1, 0, -1, 8), 3, 3)
```

```
> A
```

```
      [,1] [,2] [,3]
[1,]    1    2    0
[2,]    2    3   -1
[3,]    0   -1    8
```

# Matrix operations

```
> c <- 2
```

```
> c * A
```

```
      [,1] [,2] [,3]
[1,]    2    4    0
[2,]    4    6   -2
[3,]    0   -2   16
```

```
> B <- matrix(c(1, 7, -4, 8, 2, -5, 2, 2, 7), 3, 3)
```

```
> B
```

```
      [,1] [,2] [,3]
[1,]    1    8    2
[2,]    7    2    2
[3,]   -4   -5    7
```

```
> A + B
```

```
      [,1] [,2] [,3]
[1,]    2  10    2
[2,]    9    5    1
[3,]   -4   -6   15
```

```
> A - B
```

```
      [,1] [,2] [,3]
[1,]    0   -6   -2
[2,]   -5    1   -3
[3,]    4    4    1
```

```
> A %*% B
```

```
      [,1] [,2] [,3]
[1,]   15   12   6
[2,]   27   27   3
[3,]  -39  -42  54
```

```
> dim(A)
```

```
[1] 3 3
```

```
> det(A)
```

```
[1] -9
```

```
> A[1, 1]
```

```
[1] 1
```

```
> A[c(1, 2), c(1, 2)]
```

```
      [,1] [,2]  
[1,]    1    2  
[2,]    2    3
```

```
> A[1, ]
```

```
[1] 1 2 0
```

# Transposition & Identity

```
> t(B)
```

```
      [,1] [,2] [,3]
[1,]    1    7   -4
[2,]    8    2   -5
[3,]    2    2    7
```

```
> I <- diag(c(1, 1, 1))
```

```
> I
```

```
      [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```

# Inverse

```
> library(car)
> AI <- inv(A)
> AI
```

	[,1]	[,2]	[,3]
[1,]	-2.5555556	1.7777778	0.2222222
[2,]	1.7777778	-0.8888889	-0.1111111
[3,]	0.2222222	-0.1111111	0.1111111

```
> AI %% A
```

	[,1]	[,2]	[,3]
[1,]	1	2.775558e-17	6.661338e-16
[2,]	0	1.000000e+00	-4.440892e-16
[3,]	0	0.000000e+00	1.000000e+00

# Orthogonal vectors

```
> x <- as.vector(c(1, 2, 3))
> x

[1] 1 2 3

> y <- as.vector(c(1, 1, -1))
> y

[1] 1 1 -1

> x %*% y

      [,1]
[1,] 0

> t(x) %*% y

      [,1]
[1,] 0
```

# Orthogonal matrices

```
> X <- matrix(c(c(1, 2, 3)/sqrt(14), c(1, 1, -1)/sqrt(3), c(5,
+      -4, 1)/sqrt(42)), 3, 3)
> X
```

```
      [,1]      [,2]      [,3]
[1,] 0.2672612 0.5773503 0.7715167
[2,] 0.5345225 0.5773503 -0.6172134
[3,] 0.8017837 -0.5773503 0.1543033
```

```
> t(X) %*% X
```

```
      [,1]      [,2]      [,3]
[1,] 1.000000e+00 0.000000e+00 -2.226003e-17
[2,] 0.000000e+00 1.000000e+00 -4.809114e-17
[3,] -2.226003e-17 -4.809114e-17 1.000000e+00
```

# Eigenvalues/eigenvectors

```
> e <- eigen(A)
> e$values

[1] 8.2145852 4.0555651 -0.2701503

> e$vectors

           [,1]      [,2]      [,3]
[1,] -0.05806435 0.5357376 0.84238574
[2,] -0.20945510 0.8184906 -0.53497826
[3,] 0.97609277 0.2075052 -0.06468785
```

```
> det(A - e$values[1] * I)
[1] -2.834556e-14
> A %*% e$vectors[, 1]
      [,1]
[1,] -0.4769745
[2,] -1.7205868
[3,]  8.0181972
> e$values[1] * e$vectors[, 1]
[1] -0.4769745 -1.7205868  8.0181972
```

# Diagonalization & Rank

```
> t(e$vector) %*% A %*% e$vector
      [,1]      [,2]      [,3]
[1,] 8.214585e+00 1.071842e-15 1.820918e-16
[2,] 1.161506e-15 4.055565e+00 3.940458e-15
[3,] 1.631961e-16 3.769145e-15 -2.701503e-01
> r <- function(A) sum(svd(A)$d > 1e-15)
> r(A)
[1] 3
```

# Idempotence & Trace

```
> X <- matrix(c(1/2, 1/2, 0, 1/2, 1/2, 0, 0, 0, 1), 3, 3)
> X %*% X
      [,1] [,2] [,3]
[1,] 0.5  0.5  0
[2,] 0.5  0.5  0
[3,] 0.0  0.0  1
> sum(diag(X))
[1] 2
```

# Results

```
> eigen(X)$values
```

```
[1] 1.000000e+00 1.000000e+00 5.551115e-16
```

```
> r(X)
```

```
[1] 2
```

```
> A1 <- matrix(c(1/2, -1/2, -1/2, 1/2), 2, 2)
```

```
> A1 %*% A1
```

```
      [,1] [,2]  
[1,]  0.5 -0.5  
[2,] -0.5  0.5
```

```
> A2 <- matrix(c(1/2, 1/2, 1/2, 1/2), 2, 2)
```

```
> A2 %*% A2
```

```
      [,1] [,2]  
[1,]  0.5  0.5  
[2,]  0.5  0.5
```

```
> A1 + A2
```

```
      [,1] [,2]
[1,]    1    0
[2,]    0    1
```

```
> (A1 + A2) %% (A1 + A2)
```

```
      [,1] [,2]
[1,]    1    0
[2,]    0    1
```

```
> A1 %% A2
```

```
      [,1] [,2]
[1,]    0    0
[2,]    0    0
```

```
> A2 %*% A1
      [,1] [,2]
[1,]    0    0
[2,]    0    0
> r(A1 + A2)
[1] 2
> r(A1) + r(A2)
[1] 2
```

# Quadratic forms

```
> y <- c(1, 4, 2)
> t(y) %*% X %*% y
```

```
      [,1]
[1,] 16.5
```

```
> y <- c(3, -3, 0)
> t(y) %*% X %*% y
```

```
      [,1]
[1,]      0
```

```
> A <- matrix(c(4, -1, -1, 2), 2, 2)
```

```
> A
```

```
      [,1] [,2]  
[1,]    4  -1  
[2,]   -1    2
```

```
> AI <- inv(A)
```

```
> AI
```

```
      [,1] [,2]  
[1,] 0.2857143 0.1428571  
[2,] 0.1428571 0.5714286
```

```
> 1/A[1, 1]
```

```
[1] 0.25
```

```
> AI[1, 1] - (AI[1, 2]/AI[2, 2]) * AI[2, 1]
```

```
[1] 0.25
```