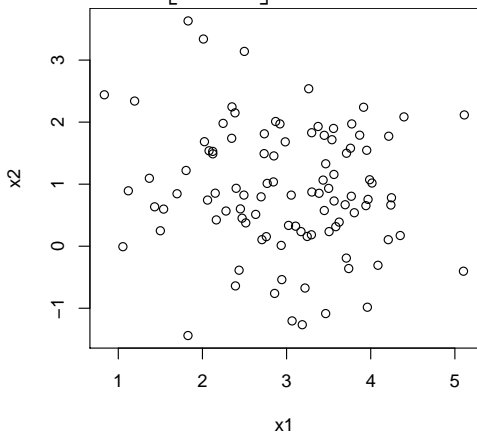


Linear Models: R Examples — Random vectors

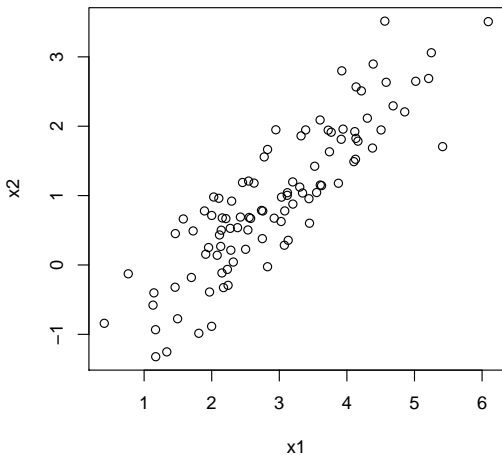
Random vectors

This is a plot of 100 samples from a distribution with mean $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$
and variance $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.



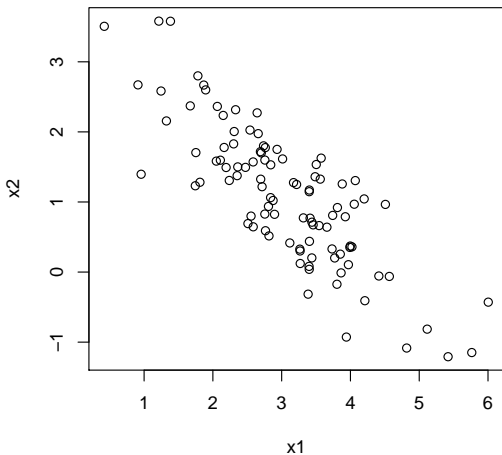
This is a plot of 100 samples from a distribution with mean $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

and variance $\begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$.



This is a plot of 100 samples from a distribution with mean $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

and variance $\begin{bmatrix} 1 & -0.8 \\ -0.8 & 1 \end{bmatrix}$.



Random quadratic forms

We use the last set of data.

```
> mu <- as.vector(c(3, 1))
> V <- matrix(c(1, -0.8, -0.8, 1), 2, 2)
> A <- matrix(c(2, -3, -1, 1), 2, 2)
> A
```

```
      [,1] [,2]
[1,]     2  -1
[2,]    -3   1
```

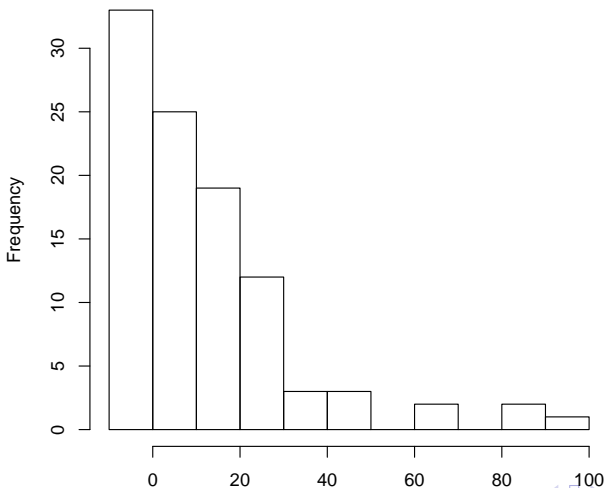
```
> sum(diag(A %*% V)) + t(mu) %*% A %*% mu
      [,1]
[1,] 13.2

> dat <- t(matrix(c(x1, x2), 100, 2))
> mean(diag(t(dat) %*% A %*% dat))

[1] 12.74784
```

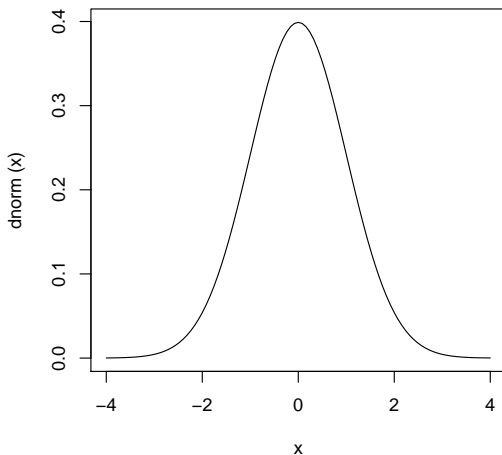
```
> hist(diag(t(dat) %*% A %*% dat))
```

Histogram of $\text{diag}(t(\text{dat}) \%*\% A \%*\% \text{dat})$



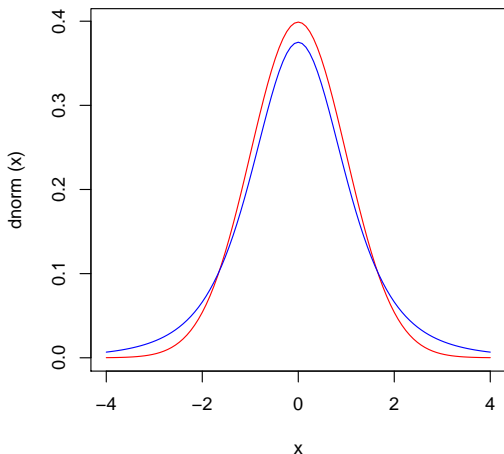
Normal distribution

```
> plot(dnorm, -4, 4)
```



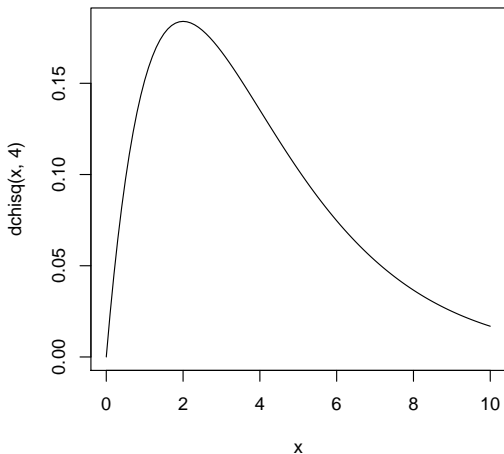
Student's- t distribution with 4 d.f.

```
> plot(dnorm, -4, 4, col = "red")  
> curve(dt(x, 4), add = TRUE, col = "blue")
```



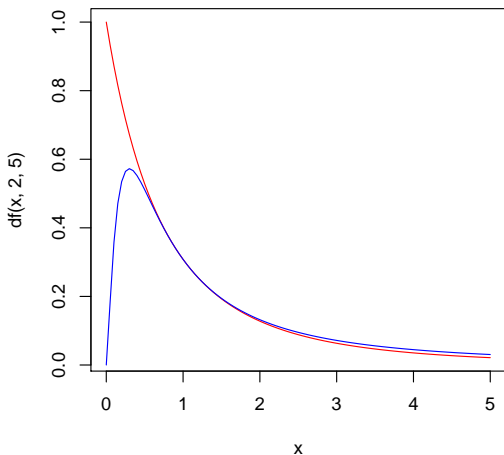
χ^2 distribution with 4 d.f.

```
> curve(dchisq(x, 4), 0, 10)
```



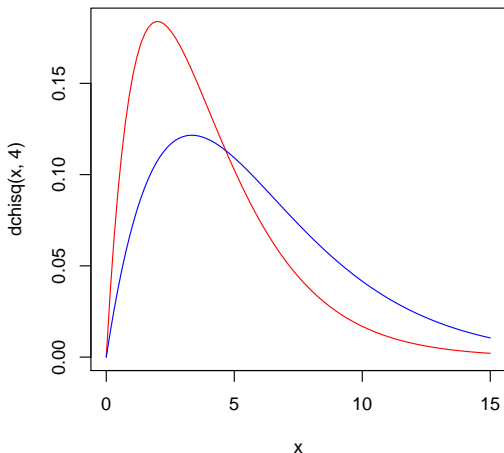
F distributions.

```
> curve(df(x, 2, 5), 0, 5, col = "red")  
> curve(df(x, 5, 2), add = TRUE, col = "blue")
```



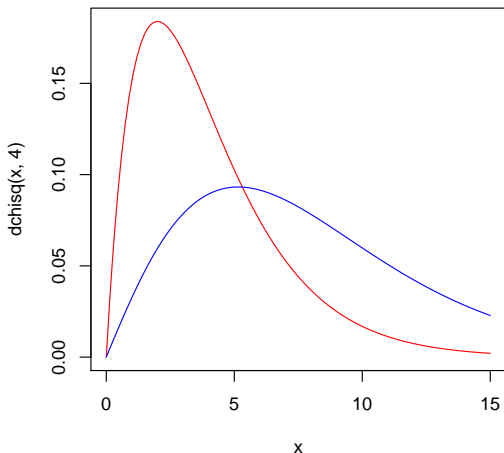
Noncentral χ^2 distribution with 4 d.f. and $\lambda = 2$

```
> curve(dchisq(x, 4), 0, 15, col = "red")  
> curve(dchisq(x, 4, 2), add = TRUE, col = "blue")
```



Noncentral χ^2 distribution with 4 d.f. and $\lambda = 4$

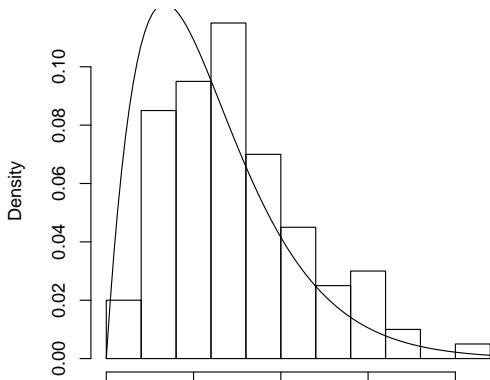
```
> curve(dchisq(x, 4), 0, 15, col = "red")  
> curve(dchisq(x, 4, 4), add = TRUE, col = "blue")
```



Generating a noncentral χ^2

```
> y <- matrix(rnorm(400, 1), 4, 100)
> hist(diag(t(y) %*% y), freq = FALSE)
> curve(dchisq(x, 4, 2), add = TRUE)
```

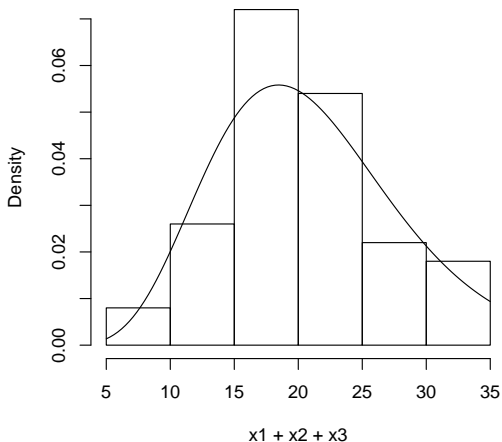
Histogram of $\text{diag}(t(y) \%*\% y)$



```
> x1 <- rchisq(100, 3, 0)
> x2 <- rchisq(100, 6, 2)
> x3 <- rchisq(100, 5, 5)
```

```
> hist(x1 + x2 + x3, freq = FALSE)
> curve(dchisq(x, 14, 7), add = TRUE)
```

Histogram of $x_1 + x_2 + x_3$



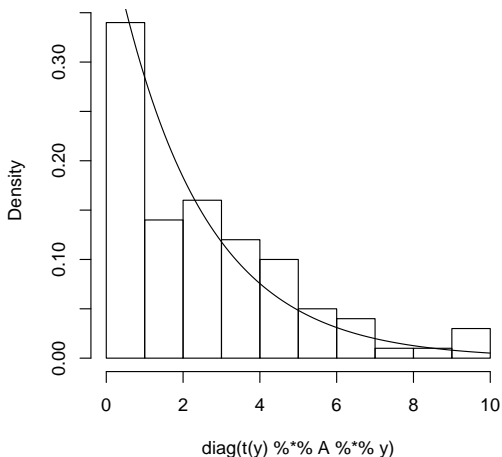
```
> A <- matrix(c(0.5, 0.5, 0, 0.5, 0.5, 0, 0, 0, 1), 3, 3)
> mu <- as.vector(c(2, -3, 0))
> lambda <- 0.5 * t(mu) %*% A %*% mu
> lambda

      [,1]
[1,] 0.25

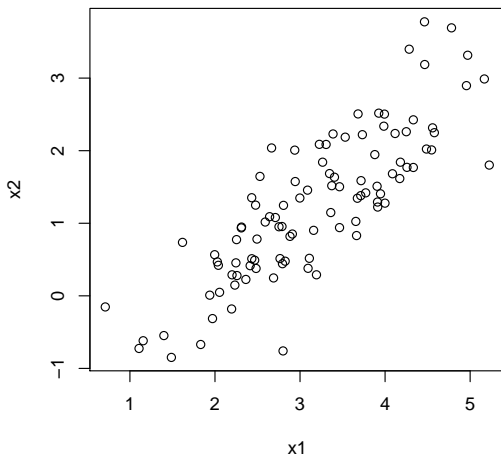
> y <- matrix(rnorm(300, mu), 3, 100)
```

```
> hist(diag(t(y) %*% A %*% y), freq = FALSE)  
> curve(dchisq(x, 2, lambda), add = TRUE)
```

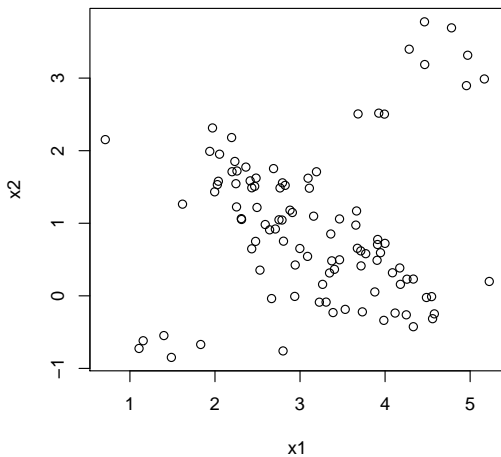
Histogram of $\text{diag}(t(y) \%*\% A \%*\% y)$



Multivariate normal distribution



Not a multivariate normal distribution



Independence of quadratic forms

```
> A <- matrix(c(1, 2, 1, 2), 2, 2)
> B <- matrix(c(2, -2, -1, 1), 2, 2)
> A %% B

      [,1] [,2]
[1,]    0    0
[2,]    0    0

> y <- matrix(rnorm(200, 0, 4), 2, 100)
> cor(diag(t(y) %% A %% y), diag(t(y) %% B %% y))

[1] -0.1162629
```

```
> plot(diag(t(y) %*% A %*% y), diag(t(y) %*% B %*% y))
```

