(73) Note $s(\underline{x}) = \bar{x}$, $\theta = \mu_F = t(F)$

$$\text{bias}_F \equiv E_F(s(\underline{x})) - t(F)$$
$$= E_F(\bar{X}) - \theta$$
$$= E_F(X) - \mu_F = \mu_F - \mu_F = 0$$

$$\text{bias}_{\hat{F}} = E_{\hat{F}}(\bar{X}) - t(\hat{F})$$
$$= E_{\hat{F}}(X) - \bar{x} \quad (X \overset{d}{=} U(x_1, x_2, \dots x_n))$$
$$= \bar{x} - \bar{x} = 0$$

(87) $$\hat{e}_0 := \frac{1}{B}\sum_{1}^{B} \exp(z_b), \quad z_b \overset{d}{=} U(0,1) \overset{d}{=} Z$$

$$\therefore \quad Var(\hat{e}_0) = \frac{1}{B} Var(e^Z)$$
$$= \frac{1}{B}[E[e^{2Z}] - (E(e^Z))^2]$$
$$= \frac{1}{B}[M_Z(2) - M_Z(1)^2],$$

where $M_Z(t)$ is m.g.f of $Z = \frac{1}{t}(e^t - 1)$

$$\therefore \quad Var(\hat{e}_0) = \frac{1}{B}\left[\frac{1}{2}(e^2-1) - (e-1)^2\right]$$
$$= \frac{1}{B}\left[-\frac{1}{2}e^2 + 2e - 3/2\right] \quad \text{—— (1)}$$

Now $$\hat{e}_1 = 1 + 0.85 + \frac{1}{B}\sum_{1}^{B}[\exp(Z_b) - (1 + 1.7Z)]$$

$$\therefore \quad Var(\hat{e}_1) = \frac{1}{B} Var[e^Z - 1.7Z]$$
$$= \frac{1}{B}\left[E([e^Z - 1.7Z]^2) - [E(e^Z - 1.7Z)]^2\right]$$

87 Contd

$$\therefore \quad Var(\hat{e}_1) = \frac{1}{B}\left[E\left[e^{2z} - 3.4ze^{z} + 1.7^2z^2\right] - \left(e-1-1.7E(z)\right)^2\right]$$

Note $E(z^2) = Var(z) + E(z)^2 = \frac{1}{12} + \left(\frac{1}{2}\right)^2 = \frac{1}{3}.$

Also $E(ze^z) = \int_0^1 ze^z \cdot 1\,dz = 1$ (integrate by parts)

$$\therefore \quad Var(\hat{e}_1) = \frac{1}{B}\left\{\frac{1}{2}(e^2-1) - 3.4 + 1.7^2/3 - (e-1.85)^2\right\}.$$

$$= \frac{1}{B}\cdot\left\{-\frac{1}{2}e^2 + 3.7e - 6.35916667\right\}$$

Thus $\dfrac{Var(\hat{e}_0)}{Var(\hat{e}_1)} = 61.3$

Note: This exercise is given in "An Introduction to the Bootstrap", Efron & Tibshirani, p 342. In the text $Var(\hat{e}_1)$ is rounded to $\frac{1}{B}\{-\frac{1}{2}e^2 + 3.7e - 6.36\}$ but this rounding actually leads to a 27% error in the reported ratio (text gives 78)

96(1) See Cityratio.r. The program estimates the parameter $\theta = \dfrac{E(1930\text{'s pop})}{E(1920\text{'s pop})}$

Bootstrap results: $t_0 = \hat{\theta} = 1.52 \quad se_B(\hat{\theta}) = 0.244$

The bootstrap replicate distribution appears skew right.

The standard normal interval is $1.52 \pm 1.645 \times 0.244 = [1.12, 1.92]$ for 90% level. Note we don't correct for bias as the norm.ci command does automatically (try norm.ci(city.boot)) as we should investigate the variability of the bias estimate before doing so.

The commands:

```
quantile(city.boot$t, 0.05)
quantile(city.boot$t, 0.95)
```

give percentile interval [1.27, 2.05] again reflecting the right skew.