

University of Melbourne
Department of Mathematics and Statistics
620-629 Integrable models
Semester 2, 2009

Lecturers J de Gier (1st half) and O Foda (2nd half)

Prerequisites 620-331 (Applied partial differential equations).

Assessment Three assignments (60%) + a final 2-hour test (40%).

Outline This is an introduction to exactly solvable nonlinear partial differential equations, from an algebraic point of view, with emphasis on the KdV and KP differential equations as representative examples.

Textbook *Solitons: Differential equations, symmetries and infinite dimensional algebras*, by T Miwa, M Jimbo and E Date (2000) Cambridge U Press.

Contents

01. *The KdV equation and its symmetries*: Symmetries and transformation groups, the Lax representation of integrable nonlinear pde's.
02. *The KdV hierarchy*: Pseudo differential operators, higher order KdV equations, infinitely many commuting symmetries, the KP hierarchy.
03. *The Hirota equation and vertex operators*: The Hirota derivative, n -soliton solutions, vertex operators, the bilinear identity.
04. *Fermion calculus*: Bosonic algebra of differentiation and multiplication, fermions, duality, charge and energy, Wick's theorem.
05. *The boson-fermion correspondence*: the normal product, realising the bosons, isomorphism of Fock spaces, realising the fermions.
06. *Transformation groups and tau functions*: Group actions and orbits, the Lie algebra $gl(\infty)$, the transformation group of the KP hierarchy.
07. *The transformation group of the KdV equation*: KP hierarchy versus KdV hierarchy, the transformation group of the KdV equation.
08. *Finite dimensional Grassmannians and Plücker relations*: Finite dimensional Grassmannians, Plücker coordinates, Plücker relations.
09. *Infinite dimensional Grassmannians*: Finite dimensional Fock spaces, the vacuum orbit, Young diagrams and character polynomials.
10. *The bilinear identity*: The bilinear identity and the Plücker relations, Plücker relations and the Hirota equation.